

Mathematics

Cycle I - versions B and C

Secondary school

Series topic	Discovering and exploring functions through motion analysis
Topics and duration	<p><u>Topic 1. Creating and interpreting graphs describing motion</u> - 2 lessons (90 minutes)</p> <p><u>Topic 2. Describing motion using graphs</u> - 1-2 lessons (45-90 minutes)</p> <p><u>Topic 3B. Discovering and exploring functions through motion analysis</u> <u>Topic 3C. Functional relationships in motion analysis</u> -2-3 lessons (90-135 minutes)</p> <p><u>Topic 4. Changing rate of change</u> -2-3 lessons (90-135 minutes)</p>
Age / Class	<p>Version B - introduction to the definition of the concept of a function (in Poland, Grade 9 - secondary school; in Slovakia, Grade 8-9).</p> <p>Version C - application of functions to motion analysis and mathematical modelling (in Poland from Grade 9 - secondary school; in Slovakia from Grades 8-9)</p>
Objective	<p>The aim of this teaching cycle is to develop a qualitative understanding of functional relationships, in particular to develop covariational reasoning and to shape an intuitive understanding of the concept of a function and a function graph, as well as the ability to use function graphs to model everyday situations related to motion.</p>

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	<ol style="list-style-type: none"> 1) Creating and interpreting graphs in the context of motion analysis 2) Developing an understanding of graphs 3) Developing an understanding of functional relationships 4) Developing co-variation reasoning 5) Discovering (version B) or reviewing (version C) the conditions defining functional relationships 6) Developing an understanding and intuition of constant and variable rates of change and functional linear and non-linear relationships
Description	<p>Students create and examine graphs describing changes in distance over time using embodied experiments. During the lesson, students use the EMPE sensor together with the EMPE software. The sensor measures the distance to the nearest obstacle, and the software shows a real-time graph of changes in this distance over time. Students are involved in embodied experiments by walking with the sensor and analysing the graphical interpretation of their movement.</p> <p>Students have the opportunity to create and observe many graphs of different shapes before introducing the definition of a function. They also perform reverse activities – they move in such a way as to reflect the movement shown in the graphs provided, and interpret and analyse various movement graphs.</p> <p>The lesson leads to an intuitive understanding of functional relationships and their interpretation in the form of function graphs. Due to the context of distance measurement and the discussion of "impossible graphs", students discover (or recall) the conditions for defining a function in a practical way and on their own, and are able to distinguish between graphs of functional and non-functional relationships.</p> <p>The conditions defining a function in the context of motion analysis are introduced (or recalled). The topic also provides the introduction of graphs showing variable rates of change (non-linear graphs) in an intuitive way.</p>
Teaching aids	<ul style="list-style-type: none"> - EMPE sensor with software - desktop computer or laptop with a web browser - projector screen - projector - worksheets for students

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In all lessons, the teacher and students use the EMPE sensor with EMPE software developed as part of the EMPE project.

Instructions for using the sensor can be found in the file available on the project website: <https://empe.uken.krakow.pl>

TOPIC 1. Creating and interpreting graphs describing motion

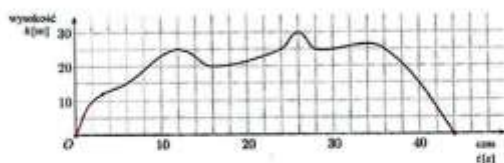
LESSON PLAN

PRE-TEST

At the beginning of the lesson cycle, we can ask students to individually and in writing complete a two-task PRETEST. Its purpose is to test their intuition in understanding graphs. The tasks from the worksheet are as follows:

Imię i nazwisko..... Klasa.....

Zadanie 1. Na wykresie zaznaczono zmiany wysokości lecącego drona nad powierzchnią ziemi w czasie trwania jego lotu. Odpowiedz na poniższe pytania.




a) Jak długo trwał ten lot?

b) Jaką maksymalną wysokość osiągnął dron?


c) Czy wykres przedstawia tor (ślad) ruchu drona? ☐ TAK ☐ NIE, ponieważ.....

d) Czy na podstawie danych z wykresu można obliczyć prędkość, z jaką poruszał się ten dron? ☐ TAK ☐ NIE, ponieważ.....


Zadanie 2. Które z rysunków mogą przedstawiać odległość piłki od bramki w pewnym czasie trwania gry.




☐ TAK ☐ NIE, ponieważ:



☐ TAK ☐ NIE, ponieważ:



☐ TAK ☐ NIE, ponieważ:



☐ TAK ☐ NIE, ponieważ:

PRE TEST

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The teacher collects the Pretests and will refer to it in subsequent stages of the lesson cycle.

Activity 1. Drama and experiment

1a) Drama. Formulating hypotheses

The teacher reads the movement scenario described in Worksheet 1:

At first, I stand still for a moment.

Then I walk at a steady, moderate pace towards the wall, and then at the same pace away from the wall. I stand still for a moment. Then I walk at a steady, fast pace towards the wall, and then at the same pace away from the wall.

I stand still for a moment.

Finally, I walk at a steady, slow pace towards the wall, and then at the same pace, I move away from the wall.

At the end, I stand still for a moment.

Then the movement described above is performed by a selected student or teacher.

It is worth starting the walk from the end of the room towards the blackboard, because in this arrangement, when the student walks from the end of the room to the beginning, we will provoke typical mistakes (see Figure 2 on the left) and it will be possible to work on them later in the lesson.

After performing the movement (drama), the teacher distributes Worksheets 1 (Figure 1) to the students.


The students' task is to make their first attempt at sketching the shape of a graph showing the changes in distance from the wall during this movement.

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Projekt współfinansowany w ramach programu Unii Europejskiej Erasmus+

Name and surname class

WORKSHEET 1

Activity 1. Sketch what the graph of my distance from the wall will look like, taking into account all stages of my movement:

I begin by standing still for a short moment. Then I walk toward the wall at a steady, moderate pace, and after reaching a point near the wall, I walk away from it at the same pace. Again, I stop and stand still for a moment. Next, I walk toward the wall at a steady, fast pace, and then walk away from the wall at the same fast pace. I pause and stand still again. Finally, I walk toward the wall at a steady, slow pace, and then walk away from it at the same slow pace. At the end, I stand still for a moment once more.

Your graph – first attempt:

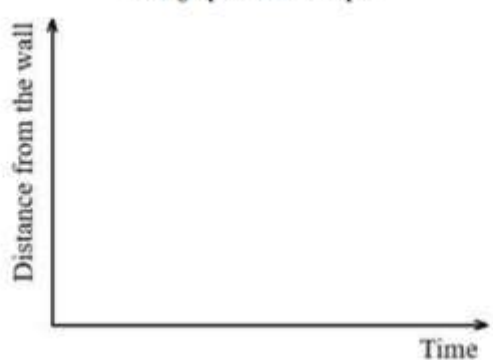


Figure1, Worksheet 1, Activity 1

1b) Discovering how the sensor works

We begin to manipulate the sensor. The teacher shows the sensor, opens the software and graph, displays it on the projector and begins the measurement. He/she directs the sensor in different directions. The students observe how the graph is created in the application. The teacher asks the question:

- What can you say about this graph? (What are the descriptions of the coordinate axes? What is being measured?)

We wait for the students' answers, which will be:

- the sensor measures the distance to the nearest obstacle it encounters,
- the graph shows changes in the distance of the sensor from the nearest obstacle over time.

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1c) Performing the experiment with the sensor. Verifying hypotheses

We perform the experiment described at the beginning of the lesson, this time using the sensor. A selected student performs the described movement independently.

Comment. Students should be instructed to hold the sensor in the same position (e.g. close to their body) and not to move it, especially in the front-back directions. It is worth performing the experiment several times, with different students.

A correctly drawn graph should have significantly different slopes of the straight lines:

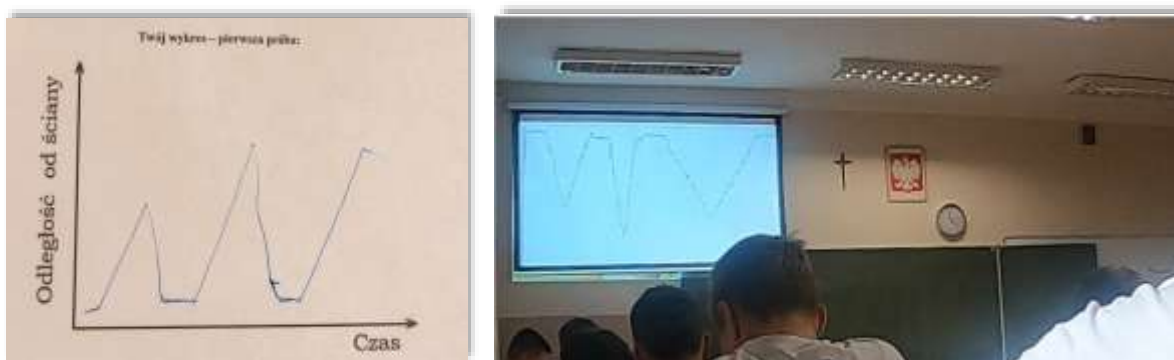


Figure2. On the left – a typical incorrect prediction by a student. On the right – a graph produced by the sensor.

Students redraw the correct graph and answer the question below the graph themselves: *What do you notice?*


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Redraw the shape of the graph made by the sensor:



What do you notice?

.....

.....

Figure3. Worksheet 1, Activity 1 (cont.).

1d) Graph analysis

Please read the students' comments.

Comment: A typical mistake is to draw the graph following the trajectory of movement to the board, as in Fig. 2 (left). We refer to this mistake in the discussion and analyse it with the pupils, explaining why such a graph is drawn incorrectly.

The teacher then asks the students questions to facilitate the analysis of the entire graph, for example:

- Why is the graph horizontal at the beginning? (we are standing, so the distance is the same)
- How does the distance between the sensor and the wall change during movement towards the wall? (the distance decreases)
- How does the distance between the sensor and the wall change during movement away from the wall? (the distance increases)
- How can we tell from the graph when we were walking fast? (greater slope of the line – more vertical segments of the line, distance covered in a short time)

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- How can we tell from the graph when we were walking slowly? (smaller slope of the straight line – more horizontal segments of the straight line, distance covered in a long time)
- Was the break in movement the same length each time?
- Was the break in movement the same distance from the board each time?
- Why are there different slopes on the graph? (Because we walk at different speeds.)
- If we walk very quickly, will the graph be vertical?
- If we walk very slowly, will the graph be horizontal? (as the pace increases, the graph becomes more and more vertical)

Comment: It is worth paying attention to distractors and imperfections in the graph. For example, in Fig. 4, you can see how the person holding the sensor stumbled while moving slowly towards the board, which is marked in red in the photograph.

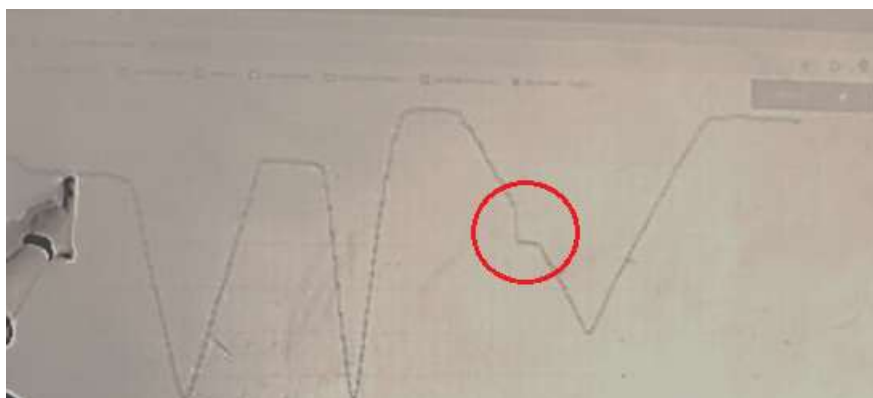


Figure4. Analysis of distractors on the graph.

Activity 2. Slowly – quickly at a steady pace TOWARDS the wall

Independent work by students. Students complete the Worksheet (p. 2):

Two people walk TOWARDS the wall, starting at the same distance from the wall, both walking at the same pace but one slowly and the other quickly. Sketch in one coordinate system what the graphs of both movements will look like.

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Person 1 walks towards the wall slowly – line named s

Person 2 walks towards the wall quickly – line named q



In your own words, explain how you can tell from the graph when the movement towards the wall was slow and when it was fast.

Selected students read out their answers.

Activity 3. Interpretation and description of the graph

The students are given the opposite task of describing in words the movement shown in the graph:

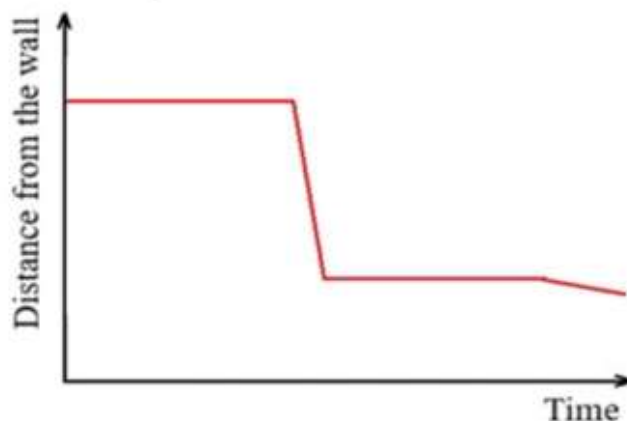
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Activity 3. The graph shows what my other movement looked like:



Describe in words what this movement might have looked like:

.....

.....

.....

.....

Figure5. Worksheet 1, Activity 3.

Activity 4. Slowly – quickly at a steady pace AWAY FROM the wall

Independent work by students. Students complete the Worksheet:

Two people walk AWAY FROM the wall, starting at the wall and ending the movement at the same distance from the wall, both walking at the same pace but one slowly and the other quickly. Sketch in one coordinate system what the graphs of both movements will look like.

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Person 1 walks slowly away from the wall – line named s
Person 2 walks quickly away from the wall – line named q



Write down in your own words how you can tell from the graph when the movement towards the wall was slow and when it was fast.

Activity 5. Summary - Graph analysis

Students fill in the table interpreting the movement graph:

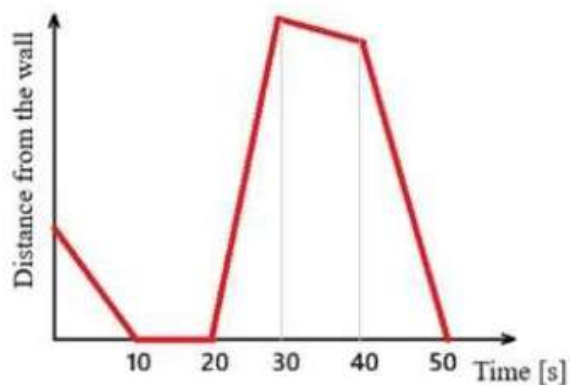
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Activity 5. Based on the graph describing the movement, complete the table (write a word or tick ✓).



Time interval	0-10 [s]	10-20 [s]	20-30 [s]	30-40 [s]	40-50 [s]
Slow/ Fast /Moderate					
TOWARDS / FROM wall					
Fastest					
Does not change distance					

How do you know when the movement was fastest?

.....

.....

Figure6. Worksheet 2, Activity 5.

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TOPIC 2. Describing motion using graphs

LESSON PLAN

This lesson is experimental in nature, and the measurements in the first part are to be taken outside the classroom (on the stairs).

Activity 6. Task about stairs

Activity 6a)

Individual work. Ask the students to solve *the staircase task* (Fig. 7) from Worksheet 3 on their own. The task is designed to test their intuitive understanding of graphs.

Staircase task:

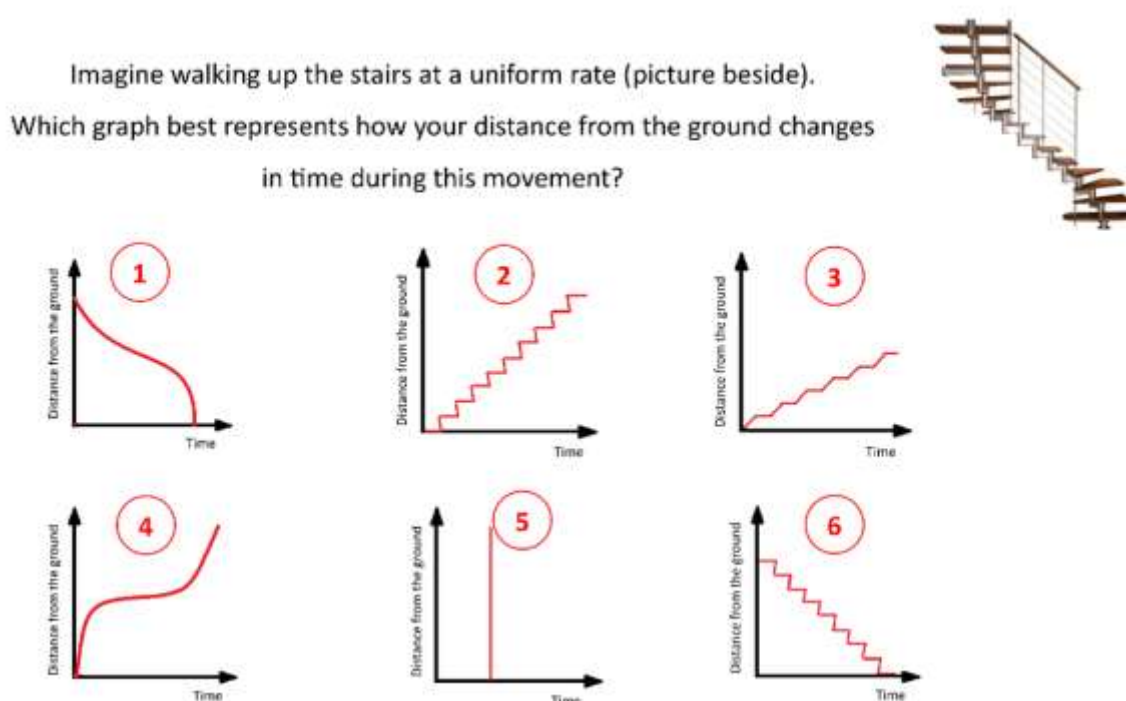


Figure7. Worksheet 3, Activity 6, Staircase task.

Activity 6b)

We conduct an experiment of walking up stairs while measuring the distance from the ground with a sensor. A laptop will be required. We record the course of the experiment.

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Comment. During the experiment, walk up the stairs very slowly and at a steady pace. Depending on the construction of the stairs, it is sometimes easier to measure the distance from the ceiling. In this case, discuss with the students how the distance from the ground and from the ceiling will change when climbing the stairs and when descending them.

Students should discover that the graph of the distance from the floor when walking up the stairs will be similar to the graph of the distance from the ceiling when walking down the stairs.

Activity 6c) Analysis of walking up stairs

Students analyse the shape of the graph in the classroom. They give the correct answer to *the staircase task*. They explain why answer 3 is correct and why answer 4 can also be considered correct if we analyse the process of climbing only one step.

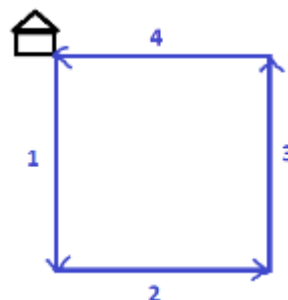
Activity 7. Task about Mr Novak

Activity 7.1

Individual work. We display the task about Mr Novak on the presentation. We ask the students to answer question a) from Worksheet 3.

Mr Novak left his house and walked around his square-shaped property at a steady pace, as shown in the picture: Solve the following tasks.

a) How did his distance from the house change at each stage of the walk? Describe as accurately as possible.



- 1.....
- 2.....
- 3.....
- 4.....

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b) Sketch a graph that illustrates, in your opinion, Mr Novak's distance from his house during the time of the walk.



c) Design and conduct an experiment illustrating Mr Novak's movement and draw the graph created using the sensor.

Activity 7.2

The students present their designs and we carry out the one that is easiest to implement.

Comment. For example, you can draw a square on the floor in the classroom (or use the graphics on the school floor in the classroom, corridor or gym). One of the students stands at the corner holding an object (in the photo, a cushion – Figure 8), from which we measure the distance by moving along the sides of the square.

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Figure8. Taking a measurement – Mr Novak's walk.

The graph that the students should receive is shown in Figure 9.

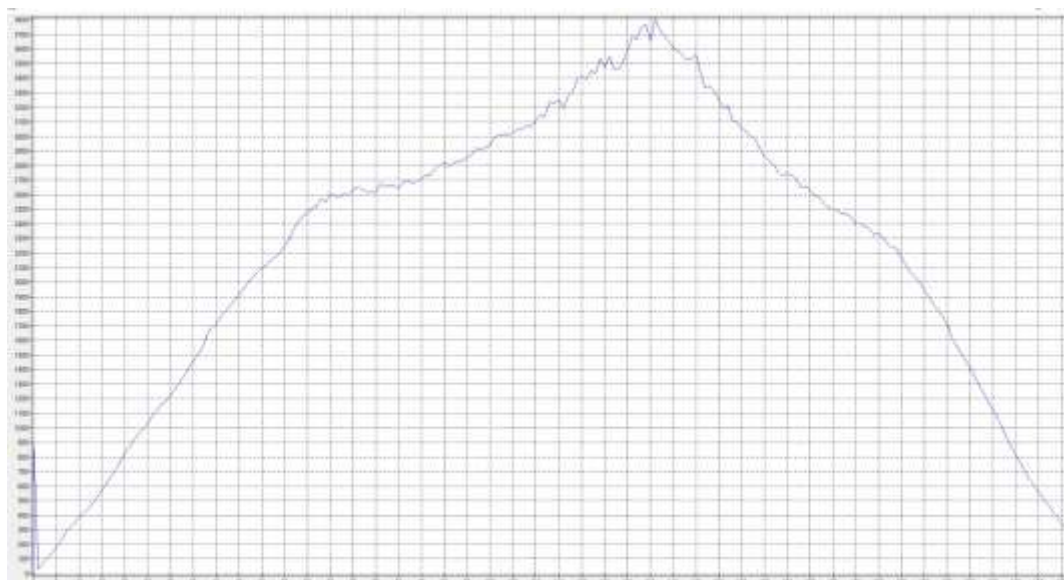


Figure9. Graph – Mr Novak's walk along the edge of the square

The students redraw the graph on their worksheets.

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Comment: At secondary school level or in gifted primary school classes, you can continue the discussion by pointing out why the graph representing changes in distance during movement along sections 2 and 3 is not linear (see scenario B).

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TOPIC 3B.

Discovering and exploring functional relationships through motion analysis

TOPIC 3C.

Functional relationships in motion analysis

LESSON PLAN

Activity 8. Interesting graphs

8a) Come up with an interesting graph

Independent work by students. Students complete Worksheet 4 by sketching an interesting graph showing their movement in relation to the wall:

Name and surname class.....

WORKSHEET 4

Activity 8. Interesting graphs

Activity 8a) Come up with an interesting graph that shows your movement relative to the wall:




Figure10. Worksheet 4, Activity 6a.

8b) Is the graph possible to perform?

Work in pairs. Students work in pairs and exchange their worksheets. Their task is to analyse their partner's graph. Students should answer the question on Worksheet 4:

Does this graph show a movement that is possible to perform?

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YES / NO / I DON'T KNOW

If YES, describe this movement in words; if NO, write why; if YOU DO NOT KNOW, explain your doubts:

Activity 8b. Swap your worksheets with the person sitting next to you.

Does this graph you received from your classmate show a movement that is possible to perform?

YES / NO / I DON'T KNOW

If YES, describe the movement in words;

if NO, write why;

if YOU DON'T KNOW, explain your doubts:

.....

.....

Figure 11. Worksheet 4. Activity 8b

8c) We perform selected graphs

Work with the whole class.

If time and organisational circumstances allow, volunteers can be asked to perform the movements shown in their classmates' graphs using the motion sensor.

8d) Analysis of impossible graphs

The teacher asks the students to redraw on the board the graphs (or parts of graphs) selected by him/her, which they believe are impossible to draw.

Various suggestions from students appear on the board. Figure 12 shows an example of a worksheet with an impossible graph, and Figure 13 shows two boards from such lessons.

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Figure12. An example worksheet with an impossible graph

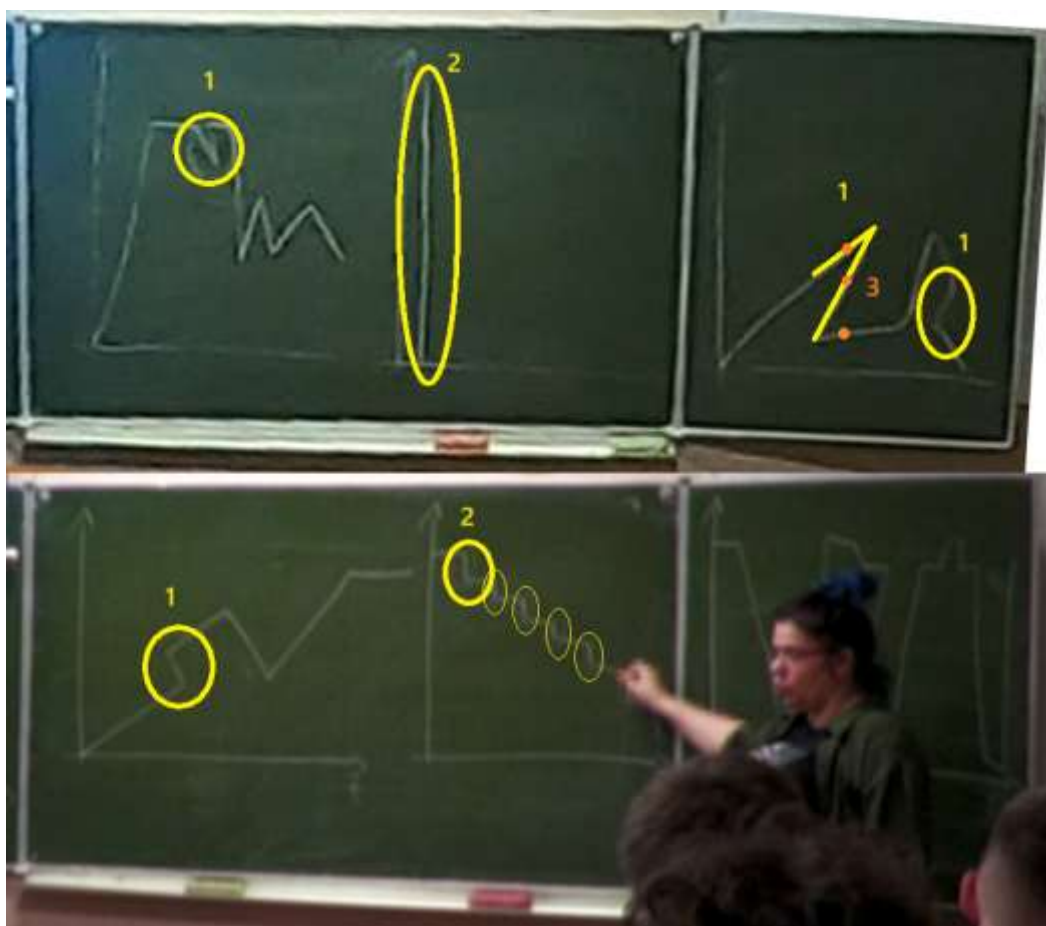


Figure13. Boards from the lesson during the analysis of impossible graphs

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The teacher ensures that graphs appear showing that both conditions defining the function are not met in different scenarios. We discuss at the board why the graph is impossible to obtain when moving with the sensor.

We take care to use natural language. There are four possible situations, which the students describe in different ways. Figure 13 shows selected situations labelled 1, 2 and 3.

In situations of type 1, students naturally say that it is impossible to go back in time (they emphasise drawing the graph "to the left"), in situations of type 2, students say that it is impossible to have so many distances at one moment, sometimes they say that it is impossible to teleport in time or have multiple locations. Situation 3 can be discussed as a consequence of situation 1, as in Figure 13, or it may arise in a different context. It is important to discuss that we cannot obtain more than 1 distance from the wall at a single moment measured by a single sensor.

INTRODUCTION OF THE DEFINITION OF FUNCTIONAL DEPENDENCIES

We summarise impossible situations.

Let us try to group the important causes of impossible graphs.

The teacher displays graphs of types of impossible graphs.

Figures 14 and 15 show an analysis of the first situation. This is usually understood by students as going back in time (Figure 14):

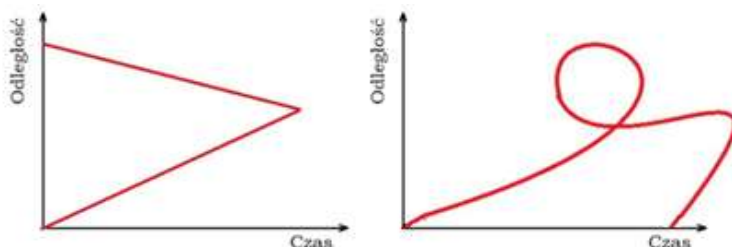


Figure 14. Analysis of the first impossible situation

We draw the students' attention to the fact that there would then be two or more distance measurements with one device at a given moment (Figure 15).

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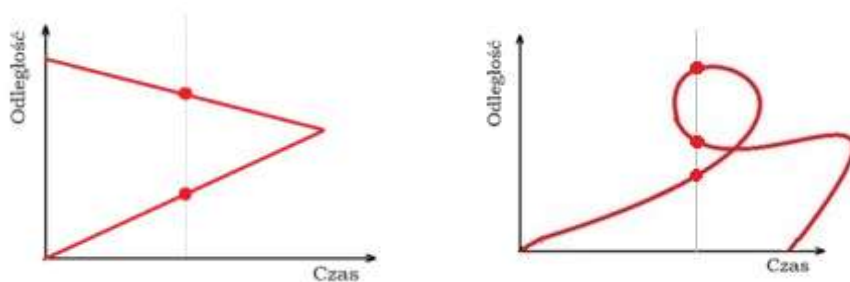


Figure15. Analysis of the first impossible situation – cont.

The next situation shown in Figure 16 shows an infinite number of measurement values at a single moment.

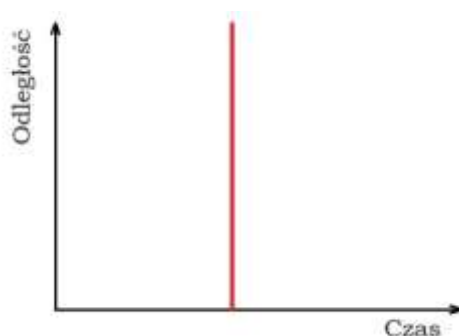


Figure16. Analysis of the second impossible situation

Comment. The vertical sections of the graph are qualitatively different for students, so we consider them separately. They do not show two or more values achieved at the same time, and the vertical line indicates continuity and movement. Students may identify such a graph with vertical upward movement. They often pay attention only to the dependent variable (y), ignoring changes in the independent variable (x). Therefore, this case should be discussed separately and only later should both cases be included in one condition.

The teacher asks the first key summary question:

- What condition must the graph meet in order to be feasible?
[Students should answer that **no more than one measurement result at a given moment.**]

The teacher asks the second key question:

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- What about this graph (Figure 17)? We do not have more than one measurement at a given moment here, so is it feasible?

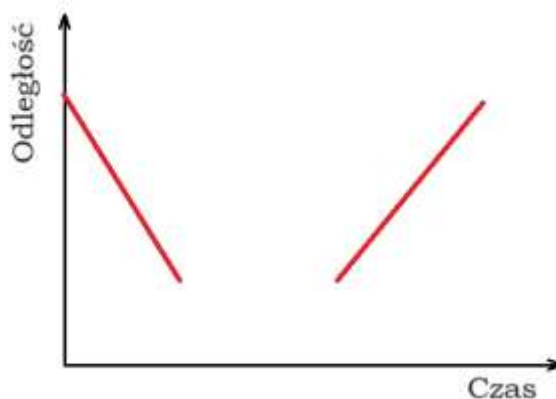


Figure17. Analysis of the last impossible situation

[Students should answer that **a break in the graph is not possible**. The teacher should ask why. If students have difficulty, the teacher may ask:

- What is our graph supposed to represent?

Students should explain that the graph shows changes in the distance between the sensor and a given obstacle over time. It is not possible for there to be no distance between the sensor and the obstacle, as the sensor will not disappear.

Comment. We also assume that the measuring sensor has not broken down. And even if it did break down, it is important to remember what it measures (the distance from the nearest obstacle), so this distance still exists and can be measured by other means.

The teacher summarises by asking the question:

- How briefly can we state the condition for the graph to be possible?

[Students should answer that **there must always be exactly one** measurement **value** at a given moment.

Comment. This condition is an intuitively formulated condition defining the functional relationship in the context of measuring distance – for each argument (point in time) there is exactly one value (measurement).

Version B: The teacher introduces:

- Relationships that satisfy this condition are called *FUNCTIONS*.

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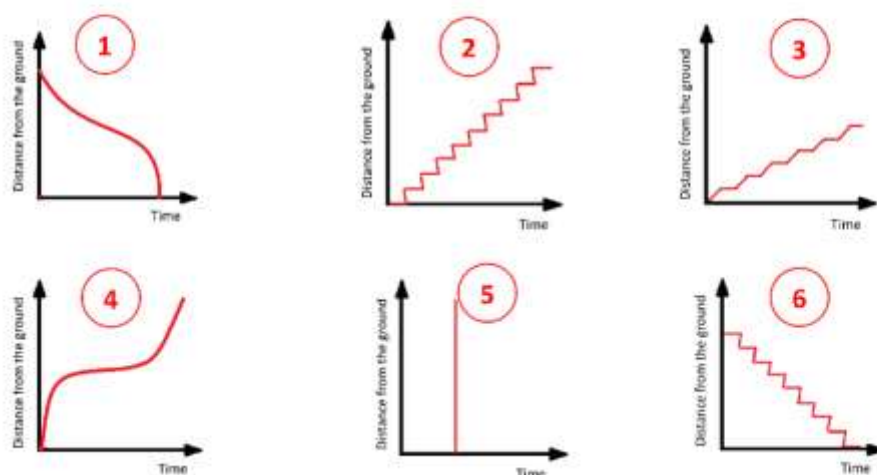
Version C:

- What are these relationships, these assignments called? [Students answer that they are functions]
- State the definition of a function.

The staircase task – recall

We recall the task from the previous lesson. The teacher displays a slide with the task.

Imagine walking up the stairs at a uniform rate (picture beside).
Which graph best represents how your distance from the ground changes in time during this movement?



[The students answer that graphs 2, 5 and 6 are not functional, and in graphs 1 and 6 the distance from the ground decreases, so they also do not correspond to the content of the task. By process of elimination, there are two that may be correct – graphs 3 and 4.]

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Activity 9. Come up with a movement whose graph is a straight horizontal line

The teacher displays the slide and the instruction.

- Come up with a movement in which the measured distance is constant:



Figure18. Come up with a movement that can be described by this graph.

The students first think about the movement individually. Then they exchange and discuss new ideas in pairs. After a while, when the pair work phase is no longer effective, the teacher leads a discussion with the whole class, collecting different ideas and asking about other types of movement. The students perform the movement they have devised, using the sensor to measure it and verify their answers.

Comment. Students first see the possibility of standing still. However, after a while, other ideas should emerge. Below are some examples that came up during the lessons.

- *Measuring the distance from the wall while jumping in place with the sensor*
- *Measuring the distance from the wall while moving at the same distance from the wall*
- *Measuring the distance from the ceiling/floor while walking freely around the room – in this case, refer to the PRE-TEST task about the drone and the fact that the drone could move horizontally and maintain a constant distance from the ground (Fig. 19)*

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Figure19. Measuring the distance from the ceiling/floor.

- *Measuring the distance from two people moving simultaneously (Fig. 20)*



Figure20. Measuring the distance from two people moving simultaneously.

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- *Measuring the distance from a person standing in the centre of a circle while moving in a circle (Fig. 21)*



Figure21Distance while moving in a circle.

- *Measuring the distance from people standing in a circle by a person spinning around their own axis (Fig. 22)*



Figure22. Distance from people standing in a circle by a person spinning around their own axis.

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In the context of various statements made by students, interesting ideas may arise that require further investigation. For example, in one class, a student suggested measuring the distance from the intersection of the diagonals in a square to the sides of the square. A model was made in which the sensor was placed on a swivel chair (Figs. 23 and 24) and the graph was checked:



Figure 23. Model for measuring the distance between the intersection point of the diagonals of a square and the sides of the square (1)



Figure 24. Model for measuring the distance between the intersection point of the diagonals of a square and the sides of the square (2)

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TOPIC 4. Changing rate of change

LESSON PLAN

Activity 10. Faster and faster (speeding up)

10a) Drama. Formulating hypotheses

The teacher reads the movement scenario described in Worksheet 5:

At the beginning, I stand still for a moment.

*Then I walk toward the wall, starting slowly and moving faster and faster,
and I stop for a moment.*

After that, I walk away from the wall, again starting slowly and moving faster and faster.

At the end, I stand still for a moment.

Next, the movement described above is performed by a selected student or the teacher. It is a good idea to start walking from the back of the room towards the blackboard.

After performing the movement (performing the drama), the teacher distributes Worksheets 5 to the students.

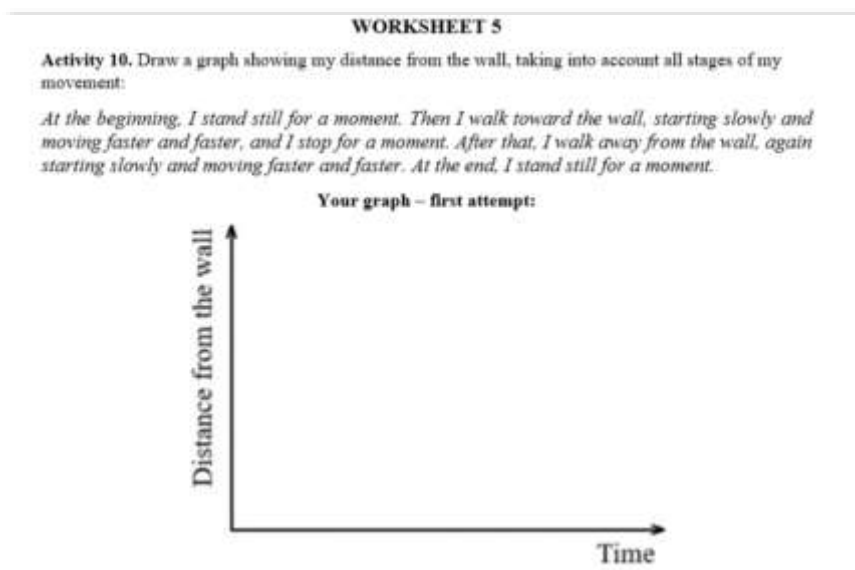


Figure25. Worksheet 5, Activity 10

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The students' task is to make their first attempt at sketching the shape of a graph showing the changes in distance from the wall during this movement.

10b) Performing the experiment with the sensor. Verifying hypotheses

We perform the experiment described at the beginning of the lesson, this time using a sensor. A selected student performs the described movement on their own.

The students redraw the correct graph and answer the question below the graph on their own: What do you notice?

LET'S CHECK with the sensor. Redraw the shape of the graph made by the sensor:



Does your first attempt match the sensor reading? YES / NO

What did you do incorrectly in your first attempt? Why do you think that happened?

.....

.....

Figure26. Worksheet 5 cont.

10c) Graph analysis

Please read the students' comments.

Comment: When analysing the graphs, we draw attention to their non-linearity and convexity/concavity.

The teacher then asks the students questions to facilitate the analysis of the entire graph, focusing on parts related to movement, for example:

- What was the speed of our movement?

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- Why is the graph not a straight line when moving to and from the board?
- How can we tell from the graph when we were walking faster and faster? (increasing "slope" of the graph – the same distances are covered in less and less time, the graph "curves")

Activity 11. Slower and slower (slowing down)

We work in a similar way to Activity 10, i.e. by proposing and verifying hypotheses using the sensor and analysing the graph of this movement. However, we do this more efficiently, using the experience gained by students in Activity 10 and their imagination. In more advanced classes, we can skip the drama exercise.

The teacher asks the student to read aloud the movement described in Activity 11:

At the beginning, I stand still for a moment.

*Then I walk toward the wall, starting quickly and moving slower and slower;
and I stop for a moment.*

After that, I walk away from the wall, again starting quickly and moving slower and slower.

At the end, I stand still for a moment.

and leads a discussion on this topic. For example, he asks the students questions:

- How does this movement scenario differ from the previous one?
- How is it similar?
- How will the distance travelled from the board change during the movement?

Then the students independently make hypotheses about the shape of the graph of this movement

and, as in Activity 10, perform this movement with the sensor to verify their hypotheses. The students then redraw the correct graph and the teacher discusses any doubts with them.

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Activity 12. Moving slower and slower / faster and faster

Activity 12a) TOWARD the wall

The teacher distributes Worksheets 6 with Activity 12. This activity is a continuation and summary of the graph analysis from the two previous activities. Independent work by students, who draw graphs for the following scenario.

Two people walk TOWARD the wall, starting from the same distance. One of them walks slower and slower (slowing down), and the other walks faster and faster (speeding up). Sketch both motions in the same coordinate system.

1. Person walking toward the wall slower and slower — line name: *sd* (slowing down)

2. Person walking toward the wall faster and faster — line name: *su* (speeding up)



In your own words, explain how you can tell from the graph when the movement towards the wall was getting slower and when it was getting faster.

Selected students read out their answers.

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Activity 12b) AWAY FROM the wall

Independent work by students who draw graphs for the following scenario.

Two people walk AWAY FROM the wall, starting from the same distance. One of them walks slower and slower (slowing down), and the other walks faster and faster (speeding up). Sketch both motions in the same coordinate system.

1. Person walking away from the wall slower and slower — line name: *sd* (slowing down)

2. Person walking away from the wall faster and faster — line name: *su* (speeding up)



Write down in your own words how you can tell from the graph when the movement away from the wall was slower and when it was faster.

Selected students read out their answers.

Activity 13. Interpretation and description of the graph

The students are given the opposite task of describing the movement shown in the graph in words (Worksheet 6):

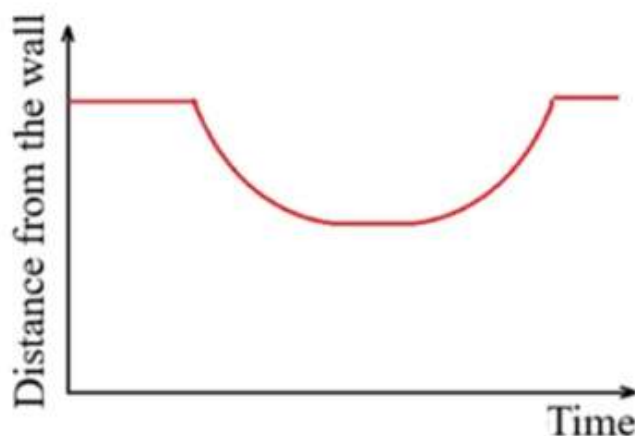
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Activity 13. The graph shows a certain movement:



Describe in words what this movement might look like:

.....

.....

.....

.....

.....

Check - perform your movement with the sensor

Does your description match the sensor's indication? YES / NO

What did you do incorrectly in your first attempt? Why do you think that was?

Figure27. Worksheet 6, Activity 13

The students' task is to describe this graph in words.

Then we perform this movement with the sensor.

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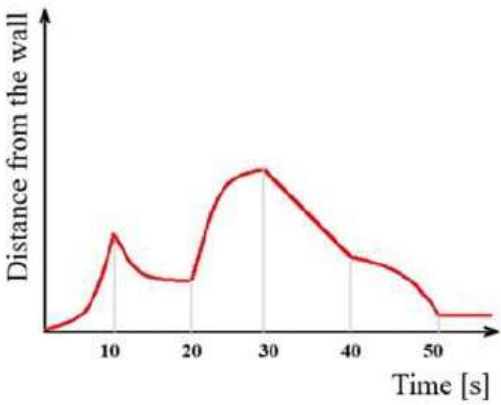
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Activity 14. Summary – Graph analysis

The students fill in the table by interpreting the movement graph:

Based on the graph describing the movement, complete the table (enter a word or tick ✓).



	0-10 [s]	10-20 [s]	20-30 [s]	30-40 [s]	40-50 [s]	after 50 [s]
Movement TOWARD /AWAY FROM the wall						
Slowing down						
Speeding up						
At a constant speed						
Does not change distance						

How can you tell when movement is getting slower and when it is getting faster?

.....

.....

.....

Figure28. Worksheet 7

Then they answer the question below the table (Figure 28):

How can you tell when the movement is getting slower and when it is getting faster?

The answers to this question and their discussion in class conclude the teaching cycle.

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POST-TEST

At the end of the lesson cycle, it is worth giving students a POST-TEST to assess their knowledge gain. The first two questions are the same as in the PRE-TEST, and the next two were discussed in class.

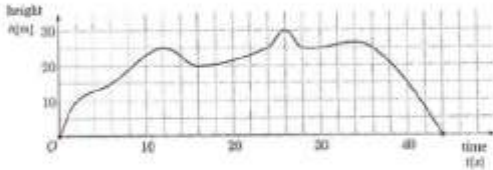


POSTTEST

Projekt współfinansowany w ramach programu Unii Europejskiej Erasmus+ 

Name and surname.....Class.....

Task 1. The graph shows changes in the height of a flying drone above the ground during its flight. Answer the following questions.

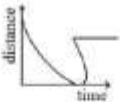


a) How long did the flight last?

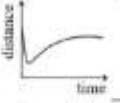
b) What was the maximum height reached by the drone?

c) Does the graph show the drone's flight path (trail)? ☐ YES ☐ NO, because.....

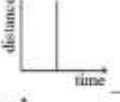
Task 2. Which of the drawings could represent the distance of the ball from the goal at a certain point in time during the game?



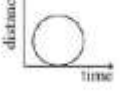
☐ YES ☐ NO, because:




☐ YES ☐ NO, because:



☐ YES ☐ NO, because:



☐ YES ☐ NO, because:



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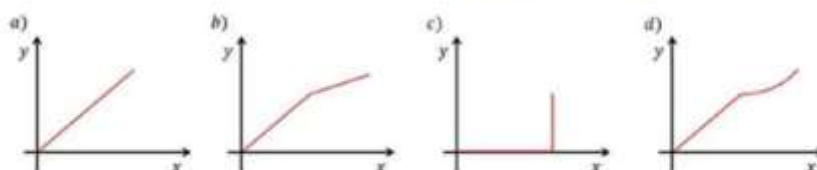


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Task 3. The diagram shows the route Kamil took from his house to the park at a constant speed. Select the graph that best describes Kamil's distance from his house during the walk.

x – time [minutes]
 y – distance from the house [meters]



Task 4. Imagine walking up the stairs at a uniform rate (picture beside). Draw a graph showing how your distance from the ground changes during this movement.



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