Projekt współfinansowany w ramach programu Unii Europejskiej Erasmus+

Mathematics
Cycle I Primary school
Understanding graphs and relationships

The scenario includes a description of adjustments for pupils with special educational needs (A, ASD, ID)

Topic 4*	**Changing rate of change** *) This topic is intended for more gifted primary school students or for use in talent development classes.
Duration	3 lessons (135 minutes)
Class/Age	The cycle is intended for pupils in the final years of primary school who are not familiar with the concept of functions, and we do not introduce this concept during the cycle (grade 7-8).
Type of adjustments	- aphasia (A), - autism spectrum disorder (ASD) - mild intellectual disabilities (ID)
Objective	*The aim of this module is to develop an **intuitive** understanding of types of relationships and their graphs.* 1) Creating and interpreting graphs in the context of movement analysis at the intuitive level 2) Developing an understanding of graphs 3) Developing an intuitive understanding of unambiguous relationships between variables 4) Developing covariational reasoning

Description	Students create and examine graphs describing changes in distance over time using embodied experiments. During the lesson, students use the EMPE sensor together with the EMPE software. The sensor measures the distance to the nearest obstacle, and the software shows a real-time graph of changes in this distance over time. Students are involved in embodiment experiments by walking with the sensor and analysing the graphical interpretation of their movement. They have the opportunity to create and observe multiple graphs of different shapes, and they also perform reverse activities – they move in such a way as to reflect the movement shown in the graphs provided, and they interpret and analyse different movement graphs. The topic leads to the introduction of graphs showing variable rates of change (non-linear graphs) in an intuitive way.
Teaching aids	- EMPE sensor with software - desktop computer or laptop with a web browser - projector screen - projector - work sheets for students

During the lesson, the teacher and students use the EMPE sensor with EMPE software developed as part of the EMPE project. Instructions for using the sensor can be found on the project website (https://empe.uken.krakow.pl).

The scenario includes a description of adjustments for pupils with special educational needs, highlighted in green. Adjustments have been made for working with pupils with **aphasia (A)**, **autism spectrum disorder (ASD)** and **mild intellectual disability (ID)**.

TOPIC 4*. Changing rate of change

LESSON PLAN

We continue the numbering of activities from Cycle I Parts 1-3.

Activity 10. Faster and faster movement - we accelerate

10a) Drama. Formulating hypotheses

The teacher reads the movement scenario described in Worksheet 5:

> *At the beginning, I stand still for a moment.*
> *Then I walk toward the wall, starting slowly and moving faster and faster,*
> *and I stop for a moment.*
> *After that, I walk away from the wall, again starting slowly and moving faster and faster.*
> *At the end, I stand still for a moment.*

The content of the instruction should be visible on the projector at all times so that students with reduced auditory perception have the opportunity for visual perception, which will help them understand the course of the experience.
Students with autism spectrum disorder (ASD), aphasia, and mild intellectual disabilities will need clarification through the teacher's demonstration of the phrase: walk faster and faster.

Then, the movement described above is performed by a selected student or teacher.

It is a good idea to involve pupils with a special educational needs certificate in the task so that they have a chance to practise this movement.

Possible difficulties during the activity;

- autism spectrum disorder (ASD):
  - in order to precisely follow the instructions in the movement scenario, the pupil may need to be given (told) the specific number of steps to take or the point to which they are to walk (this can be marked on the floor, e.g. with adhesive tape). The number of steps or the indicated point will, of course, depend on the size of the room in which the lesson takes place.
  - the student may expect (e.g. by asking questions) additional instruction,
- Aphasia (A):

- the student may have difficulty understanding the content of the movement scenario and will therefore need the help of the teacher, who will read the scenario and perform the desired movement, and only after this help will the student perform the task,

- mild intellectual disability (ID):
  - similar to a student with aphasia, they may have difficulty understanding the content of the movement scenario and will therefore need the help of a teacher who will simultaneously read the scenario and perform the desired movement, and only after this assistance will the student perform the task,
  - The pupil may need to be told (spoken) the specific number of steps they are to take or the point they are to reach (this can be marked on the floor, e.g. with adhesive tape). The number of steps or the point indicated will, of course, depend on the size of the room in which the lesson is taking place.

It is a good idea to start the walk from the back of the room, moving towards the blackboard.

After completing the movement (performing the drama), the teacher distributes Worksheets 5 to the pupils.

Students with autism spectrum disorder (ASD) complete WORKSHEET 5 (ASD).
Students with aphasia complete WORKSHEET 5 (A).
Students with mild intellectual disabilities complete WORKSHEET 5 (ID).

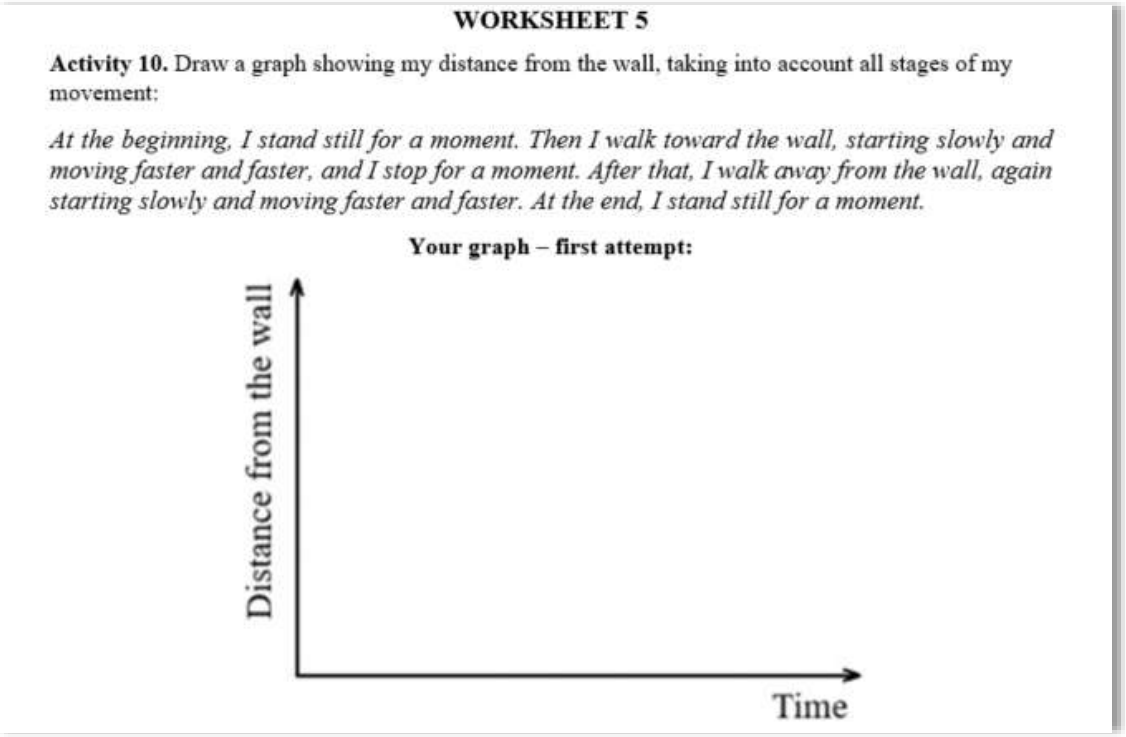


Figure1 . Worksheet 5, Activity 10

The task for students is to make their first attempt at sketching the shape of a graph showing the changes in distance from the wall during this movement.

Possible difficulties during the task:

- autism spectrum disorder (ASD):
  - the student may expect additional instruction because they may feel confused by such freedom of action,
- aphasia (A):
  - the student may have difficulty reading and understanding the movement scenario and will therefore need the help of the teacher, who will read the subsequent stages of the movement to them,
  - the student may need information about what the individual arrows (axes of the system) mean,
  - it may be necessary to show the student where to start the graph,

- mild intellectual disability (ID):
  - similar to a student with aphasia, they may have difficulty reading and understanding the movement scenario and will therefore need the help of a teacher who will read the subsequent stages of the movement to them,
  - in the case of difficulties with reading comprehension, it may be necessary to work in stages. The teacher reads a fragment of the movement scenario and the student draws it on the graph. Then the teacher reads the next fragment and the student proceeds as before. The sequence of actions is repeated until the end of the scenario.
  - The student will most likely need information about what the individual arrows (axes of the system) mean.
  - It will most likely be necessary to show the student where to start the graph.

10b) Performing the experiment with the sensor. Verifying hypotheses

We perform the experiment described at the beginning of the lesson, this time using the sensor. The selected student performs the described movement independently.

While the exercise is being performed, the teacher reads the instructions.

Comment: Students should be instructed to hold the sensor in the same position.

The pupils redraw the correct graph.

If possible, for students with autism spectrum disorder (ASD), aphasia and mild intellectual disabilities, it would be a good idea to print out the graph created during the experiment so that they can paste it into their notebooks.

Possible difficulties during the activity:

- autism spectrum disorder (ASD):
  - if the student does not have graphomotor difficulties, they should not have any major difficulties with this task,
  - if graphomotor difficulties occur, the teacher's help will be necessary,
- Aphasia (A):
  - as the student may make significant mistakes when redrawing the graph, it is necessary for the teacher to monitor their actions and provide assistance if necessary,
- mild intellectual disability (ID):

- similar to a student with aphasia, they may make significant mistakes when redrawing the graph, and supervision and assistance from the teacher will be necessary.

The pupils answer the question below the graph themselves: *What do you notice?*

Since we are interested in all possible answers from students, we deliberately ask an imprecise question. With such a high degree of generality, pupils with aphasia and intellectual disabilities may not be able to answer this question. However, if they decide to answer (in writing), they should be helped to formulate their thoughts by suggesting appropriate words or asking supporting questions.

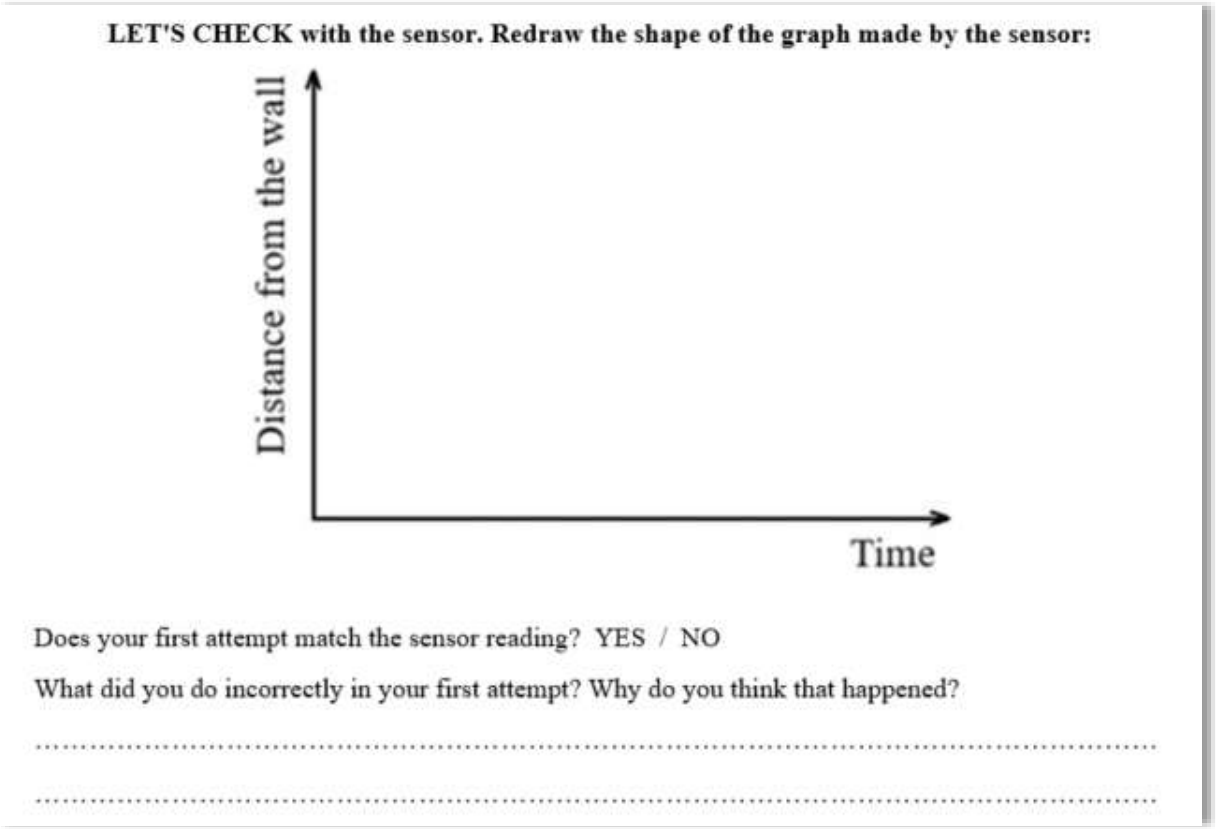


Figure2 . Worksheet 5 cont.

10c) Analysis of the graph

Please read the students' statements.

> *Comment: When analysing the graphs, we draw attention to their non-linearity and convexity/concavity.*

The teacher then asks the pupils questions to facilitate the analysis of the entire graph, focusing on parts related to movement, for example:

- What was the speed of our movement?

- Why is the graph not a straight line when moving to and from the board?

- How can we tell from the graph when we were walking faster and faster? (increasing "slope" of the graph – the same distances are covered in less and less time, the graph "curves")

The relevant section should be indicated on the graph. Students with autism spectrum disorder (ASD), aphasia and mild intellectual disability with aphasia may be asked to mark this section of the graph with a selected colour. Under the graph, the statement: increasing slope of the graph should be underlined with the same colour. After completing the task, check that it has been done correctly.

Activity 11. Slower and slower movement – we are slowing down

We work in a similar way to Activities 10, i.e. by making and verifying hypotheses using the sensor and analysing the graph of this movement. However, we do this more efficiently, using the experience gained by the pupils in Activity 10 and their imagination. In more advanced classes, we can skip the drama.

The teacher asks the student to read aloud the movement described in Activity 11:

> *At the beginning, I stand still for a moment.*
>
> *Then I walk toward the wall, starting quickly and moving slower and slower,*
>
> *and I stop for a moment.*
>
> *After that, I walk away from the wall, again starting quickly and moving slower and slower.*
>
> *At the end, I stand still for a moment.*

The content of the instruction should be visible on the projector at all times so that students with reduced auditory perception have the opportunity for visual perception, which will help them understand the course of the experience.
Students with autism spectrum disorder (ASD), aphasia, and mild intellectual disabilities will need clarification through the teacher demonstrating the phrase: walk slower and slower.

The teacher leads a discussion on this topic. For example, they ask the pupils questions such as:

- How does this movement scenario differ from the previous one?

- How is it similar?

- How will the distance travelled from the board change during the movement?

The pupils then independently formulate hypotheses about the shape of the graph for this movement and, as in Activity 10, perform this movement with the sensor to verify their hypotheses. The students then redraw the correct graph and the teacher discusses any doubts with them.

It is a good idea to involve pupils with special educational needs in this task so that they have a chance to practise this movement.

Possible difficulties during the activity;

- autism spectrum disorder (ASD):
  - in order to precisely follow the instructions in the movement scenario, the student may need to be given (told) the specific number of steps to take or the point to which they are to go (this can be marked on the floor, e.g. with adhesive tape). The number of steps or the indicated point will, of course, depend on the size of the room in which the lesson takes place.
  - the student may expect (e.g. by asking questions) additional instruction,
- Aphasia (A):
  - the student may have difficulty understanding the content of the movement scenario and will therefore need the help of the teacher, who will read the scenario and perform the desired movement, and only after this help will the student perform the task,
- mild intellectual disability (ID):
  - similar to a student with aphasia, they may have difficulty understanding the content of the movement scenario and will therefore need the help of a teacher who will

simultaneously read the scenario and perform the desired movement ( ), and only after this assistance will the student perform the task,
- the pupil may need to be told (spoken) the specific number of steps to be taken or the point to be reached (this can be marked on the floor, e.g. with adhesive tape). The number of steps or the point to be reached will, of course, depend on the size of the room in which the lesson is taking place.

Activity 12. Moving slower and slower/faster and faster

Activity 12a) TOWARD the wall

The teacher distributes Worksheets 6 with Activity 12. This activity is a continuation and summary of the analysis of the graph from the two previous activities. Independent work by students who draw graphs for the following scenario

Two people walk TOWARD the wall, starting from the same distance. One of them walks slower and slower (slowing down), and the other walks faster and faster (speeding up). Sketch both motions in the same coordinate system.

Use two different colours to draw both graphs. Ensure that students with autism spectrum disorder (ASD), aphasia and mild intellectual disabilities perform the task (graph) well through ongoing monitoring and additional instruction.

1. Person walking toward the wall slower and slower — line name: *sd (slowing down)*

2. Person walking toward the wall faster and faster — line name: *su (speeding up)*

Describe in your own words how you can tell from the graph when the movement towards the wall was getting slower and when it was getting faster.

Students with aphasia and students with mild intellectual disabilities should try to complete this task on their own. However, they may not be able to analyse the graph without the teacher's help.

- Students with aphasia may have difficulty expressing their thoughts in writing.
- For pupils with mild intellectual disabilities, the task is difficult because it requires them to perform a series of mental operations and formulate oral statements at the same time.

Selected students read out their statements.

Activity 12b) AWAY FROM the wall

Independent work by students who draw graphs for the following scenario.

Two people walk away from the wall, starting from the same distance. One of them walks slower and slower (slowing down), and the other walks faster and faster (speeding up).Sketch both motions in the same coordinate system.

As in Activity 12a:

use two different colours to draw both graphs. Ensure that students with autism spectrum disorder (ASD), aphasia and mild intellectual disabilities perform the task (graph) well through ongoing monitoring and additional instruction.

1. Person walking away from the wall slower and slower — line name: *sd (slowing down)*

2. Person walking away from the wall faster and faster — line name: *su (speeding up)*

In your own words, how can you tell from the graph when the movement from the wall was getting slower and when it was getting faster?

As in activity 12a:

Students with aphasia and students with mild intellectual disabilities should try to complete this task on their own. However, they may not be able to analyse the graph without the teacher's help.

- Students with aphasia may have difficulty expressing their thoughts in writing.
- For pupils with mild intellectual disabilities, the task is difficult because it requires them to perform a series of mental operations and formulate oral statements at the same time.

Selected students read out their statements.

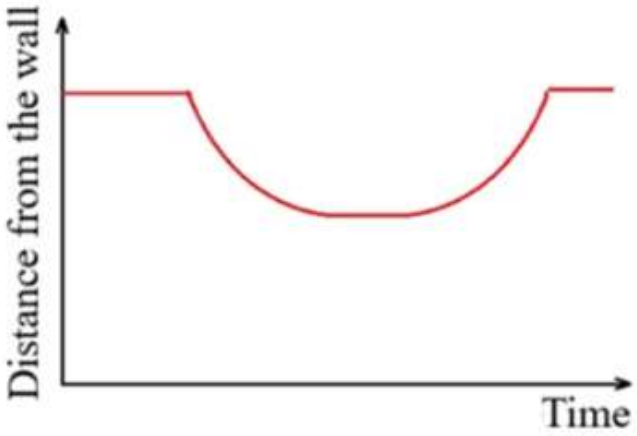
Activity 13. Interpretation and description of the graph

The pupils are asked to perform the reverse activity, which consists of describing the movement shown in the graph in words (Worksheet 6).

Students with autism spectrum disorder (ASD), aphasia, and mild intellectual disabilities may have difficulty interpreting the graph on their own. Constant supervision and possible assistance from the teacher will be necessary. You can suggest that they mark the individual parts of the graph with consecutive numbers (1–3) and then use these numbers to describe the movement shown. It will be helpful to include a plan on the worksheet.

The task for the students is to describe this graph in words.

Activity 13. The graph shows a certain movement:



Describe in words what this movement might look like:

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.....

.....

.....

.....

.....

Check - perform your movement with the sensor

Does your description match the sensor's indication? YES / NO

What did you do incorrectly in your first attempt? Why do you think that was?

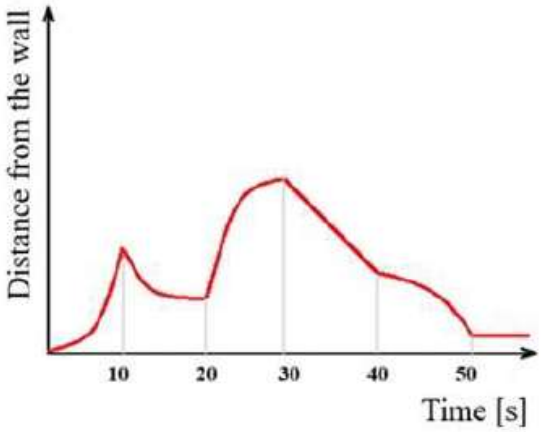
Figure3 Activity 13, Worksheet 6

Then we perform this movement with the sensor.

Activity 14 Summary – Graph analysis

Students fill in the table interpreting the movement graph:

Based on the graph describing the movement, complete the table (enter a word or tick ✓).



	0-10 [s]	10-20 [s]	20-30 [s]	30-40 [s]	40-50 [s]	after 50 [s]
Movement TOWARD /AWAY FROM the wall						
Slowing down						
Speeding up						
At a constant speed						
Does not change distance						

How can you tell when movement is getting slower and when it is getting faster?

.....

.....

.....

Figure3 . Worksheet 7

Then they answer the question below the table (Figure 4).

How can you tell when the movement is getting slower and when it is getting faster?

Students with autism spectrum disorder (ASD), aphasia and mild intellectual disabilities may have difficulty interpreting the graph and completing the table correctly on their own. They will

need help and guidance not only through additional verbal instruction, but also through detailed written instructions. It will be helpful to specify that the first row of the table should contain the selected word, and the remaining rows should contain the letter V.
When working with the table, pupils with aphasia and pupils with mild intellectual disabilities will find it helpful to number the rows and cover the rows they are not analysing at a given moment.
However, constant supervision and possible assistance from the teacher will be necessary.
At the end of the lesson, make sure that the pupils have taken proper notes.

The answers to this question and their discussion in class conclude the teaching cycle.

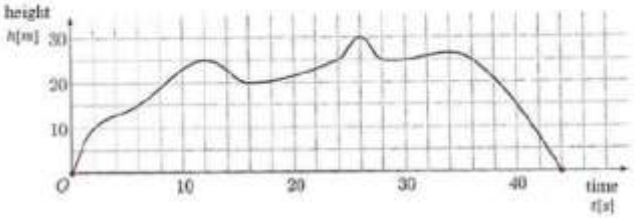
POST-TEST

At the end of the teaching cycle, it is worth offering students a POST-TEST to assess their knowledge gain. The tasks are the same as in the PRE-TEST.

POSTTEST

Name and surname.....**Class**.....

Task 1. The graph shows changes in the height of a flying drone above the ground during its flight. Answer the following questions.



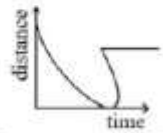
a) How long did the flight last? .....

b) What was the maximum height reached by the drone? .....

c) Does the graph show the drone's flight path (trail)? ○YES ○NO,

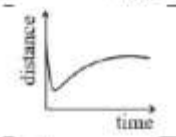
because.....

Task 2. Which of the drawings could represent the distance of the ball from the goal at a certain point in time during the game?



○YES ○NO , because:

.....

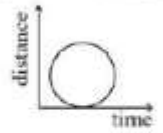


○YES ○NO , because:

.....

○YES ○NO , because:

.....



○YES ○NO , because:

.....

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