Projekt współfinansowany w ramach programu Unii Europejskiej Erasmus+

Mathematics
Cycle I Primary school
Understanding graphs and relationships

Topic 4*	**Changing rate of change** *) This topic is intended for more gifted primary school students or for use in talent development classes.
Duration	3 lessons (135 minutes)
Class/Age	The cycle is intended for pupils in the final years of primary school who are not familiar with the concept of functions, and we do not introduce this concept during the cycle (grade 7-8).
Objective	*The aim of this module is to develop an **intuitive** understanding of types of relationships and their graphs.* 1) Creating and interpreting graphs in the context of movement analysis at the intuitive level 2) Developing an understanding of graphs 3) Developing an intuitive understanding of unambiguous relationships between variables 4) Developing covariational reasoning
Description	Students create and examine graphs describing changes in distance over time using embodied experiments. During the lesson, students use the EMPE sensor together with the EMPE software. The sensor measures the distance to the nearest obstacle, and the software shows a real-time graph of changes in this distance over time. Students are involved in embodiment experiments by walking with the sensor and analysing the graphical interpretation of their movement. They have the opportunity to create and observe multiple graphs of different shapes, and they also perform reverse activities – they move in such a way as to reflect the movement shown in the graphs provided, and they interpret and analyse different movement graphs. The topic leads to the introduction of graphs showing variable rates of change (non-linear graphs) in an intuitive way.

Teaching aids	- EMPE sensor with software - desktop computer or laptop with a web browser - projector screen - projector - work sheets for students

During the lesson, the teacher and students use the EMPE sensor with EMPE software developed as part of the EMPE project. Instructions for using the sensor can be found on the project website (https://empe.uken.krakow.pl).

TOPIC 4*. Changing rate of change

LESSON PLAN

We continue the numbering of activities from Cycle I Parts 1-3.

Activity 10. Faster and faster (speeding up)

10a) Drama. Formulating hypotheses

The teacher reads the movement scenario described in Worksheet 5:

> *At the beginning, I stand still for a moment.*
> *Then I walk toward the wall, starting slowly and moving faster and faster,*
> *and I stop for a moment.*
> *After that, I walk away from the wall, again starting slowly and moving faster and faster.*
> *At the end, I stand still for a moment.*

Then, the movement described above is performed by a selected student or teacher.

It is a good idea to start the walk from the back of the room, moving towards the blackboard.

After completing the movement (performing the drama), the teacher distributes Worksheets 5 to the students.

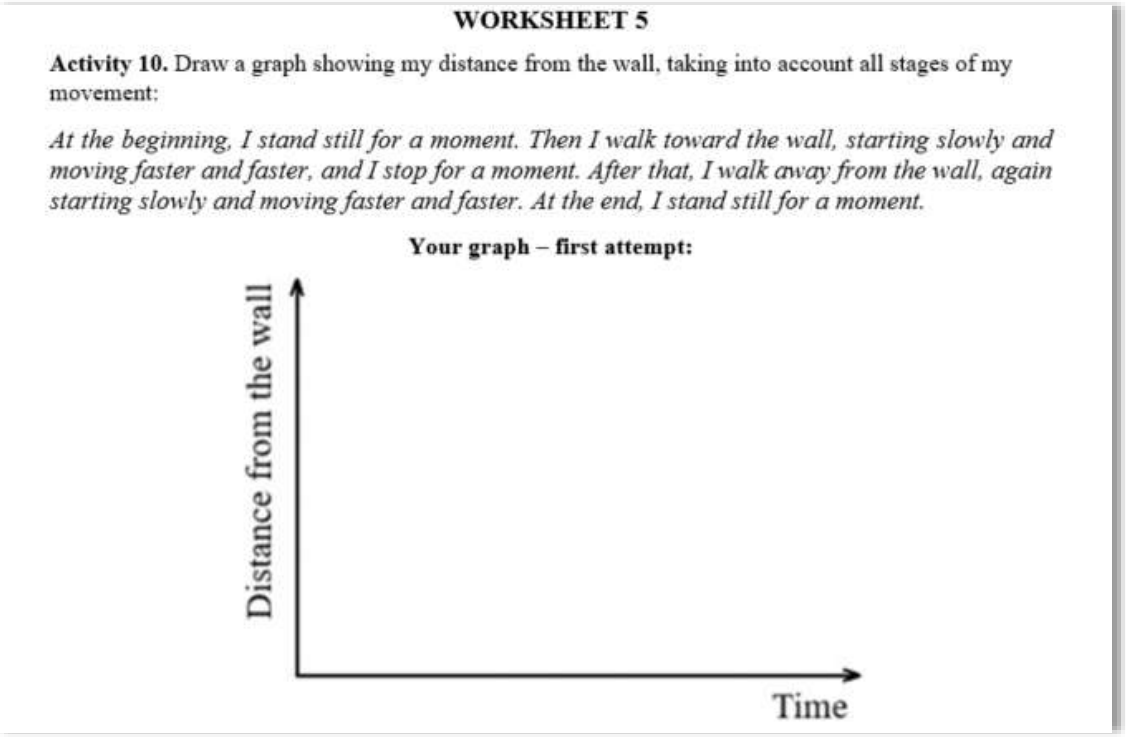


Figure1 . Worksheet 5, Activity 10

The task for students is to make their first attempt at sketching the shape of a graph showing the changes in distance from the wall during this movement.

10b) Performing the experiment with the sensor. Verifying hypotheses

We perform the experiment described at the beginning of the lesson, this time using the sensor. The selected student performs the described movement independently.

Comment: Students should be instructed to hold the sensor in the same position.

The pupils redraw the correct graph.

The pupils answer the question below the graph themselves: *What do you notice?*

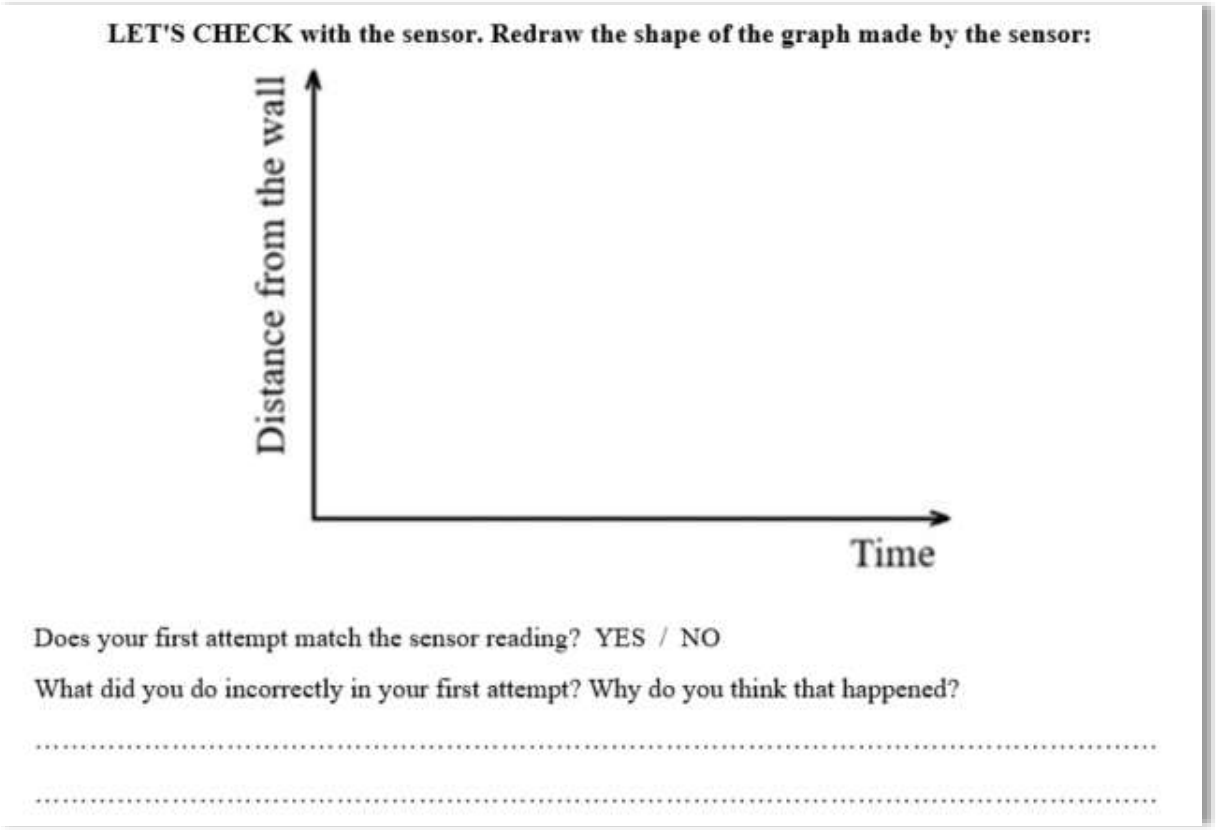


Figure2 . Worksheet 5 cont.

10c) Analysis of the graph

Please read the students' statements.

> *Comment: When analysing the graphs, we draw attention to their non-linearity and convexity/concavity.*

The teacher then asks the pupils questions to facilitate the analysis of the entire graph, focusing on parts related to movement, for example:

- What was the speed of our movement?

- Why is the graph not a straight line when moving to and from the board?

- How can we tell from the graph when we were walking faster and faster? (increasing "slope" of the graph – the same distances are covered in less and less time, the graph "curves")

Activity 11. Slower and slower (slowing down)

We work in a similar way to Activities 10, i.e. by making and verifying hypotheses using the sensor and analysing the graph of this movement. However, we do this more efficiently, using the experience gained by the pupils in Activity 10 and their imagination. In more advanced classes, we can skip the drama.

The teacher asks the student to read aloud the movement described in Activity 11:

> *At the beginning, I stand still for a moment.*
>
> *Then I walk toward the wall, starting quickly and moving slower and slower,*
>
> *and I stop for a moment.*
>
> *After that, I walk away from the wall, again starting quickly and moving slower and slower.*
>
> *At the end, I stand still for a moment.*

The teacher leads a discussion on this topic. For example, they ask the pupils questions such as:

- How does this movement scenario differ from the previous one?

- How is it similar?

- How will the distance travelled from the board change during the movement?

The pupils then independently formulate hypotheses about the shape of the graph for this movement and, as in Activity 10, perform this movement with the sensor to verify their hypotheses. The students then redraw the correct graph and the teacher discusses any doubts with them.

Activity 12. Moving slower and slower/faster and faster

Activity 12a) TOWARD the wall

The teacher distributes Worksheets 6 with Activity 12. This activity is a continuation and summary of the analysis of the graph from the two previous activities. Independent work by students who draw graphs for the following scenario

Two people walk TOWARD the wall, starting from the same distance. One of them walks slower and slower (slowing down), and the other walks faster and faster (speeding up). Sketch both motions in the same coordinate system.

1. Person walking toward the wall slower and slower — line name: *sd (slowing down)*

2. Person walking toward the wall faster and faster — line name: *su (speeding up)*

Describe in your own words how you can tell from the graph when the movement towards the wall was getting slower and when it was getting faster.

Selected students read out their statements.

Activity 12b) AWAY FROM the wall

Independent work by students who draw graphs for the following scenario.

Two people walk AWAY FROM the wall, starting from the same distance. One of them walks slower and slower (slowing down), and the other walks faster and faster (speeding up).Sketch both motions in the same coordinate system.

1. Person walking away from the wall slower and slower — line name: *sd (slowing down)*

2. Person walking away from the wall faster and faster — line name: *su (speeding up)*

In your own words, how can you tell from the graph when the movement from the wall was getting slower and when it was getting faster?

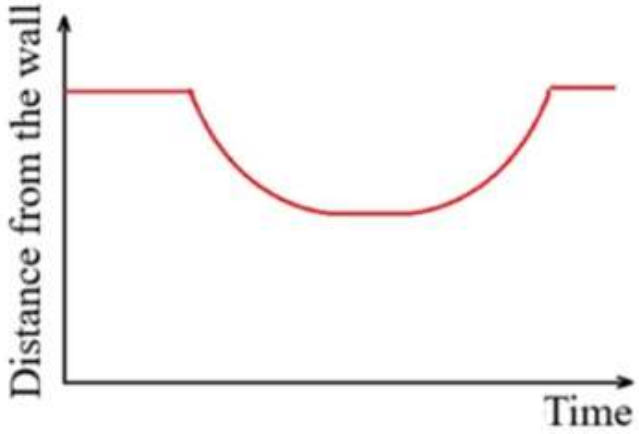
Selected students read out their statements.

Activity 13. Interpretation and description of the graph

The pupils are asked to perform the reverse activity, which consists of describing the movement shown in the graph in words (Worksheet 6).

The task for the students is to describe this graph in words.

Activity 13. The graph shows a certain movement:



Describe in words what this movement might look like:

.....

.....

.....

.....

.....

.....

Check - perform your movement with the sensor

Does your description match the sensor's indication? YES / NO

What did you do incorrectly in your first attempt? Why do you think that was?

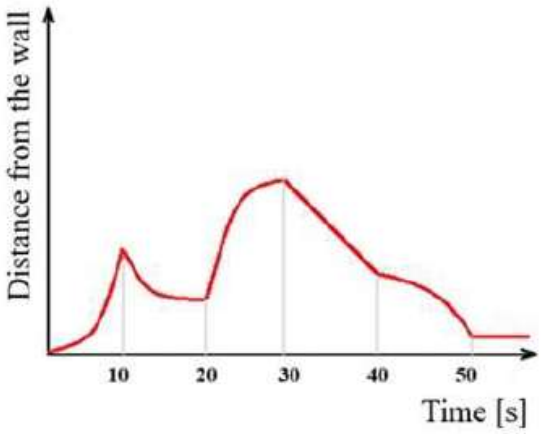
Figure3 Activity 13, Worksheet 6

Then we perform this movement with the sensor.

Activity 14 Summary – Graph analysis

Students fill in the table interpreting the movement graph:

Based on the graph describing the movement, complete the table (enter a word or tick ✓).



	0-10 [s]	10-20 [s]	20-30 [s]	30-40 [s]	40-50 [s]	after 50 [s]
Movement TOWARD /AWAY FROM the wall						
Slowing down						
Speeding up						
At a constant speed						
Does not change distance						

How can you tell when movement is getting slower and when it is getting faster?

.....

.....

.....

Figure4 . Worksheet 7

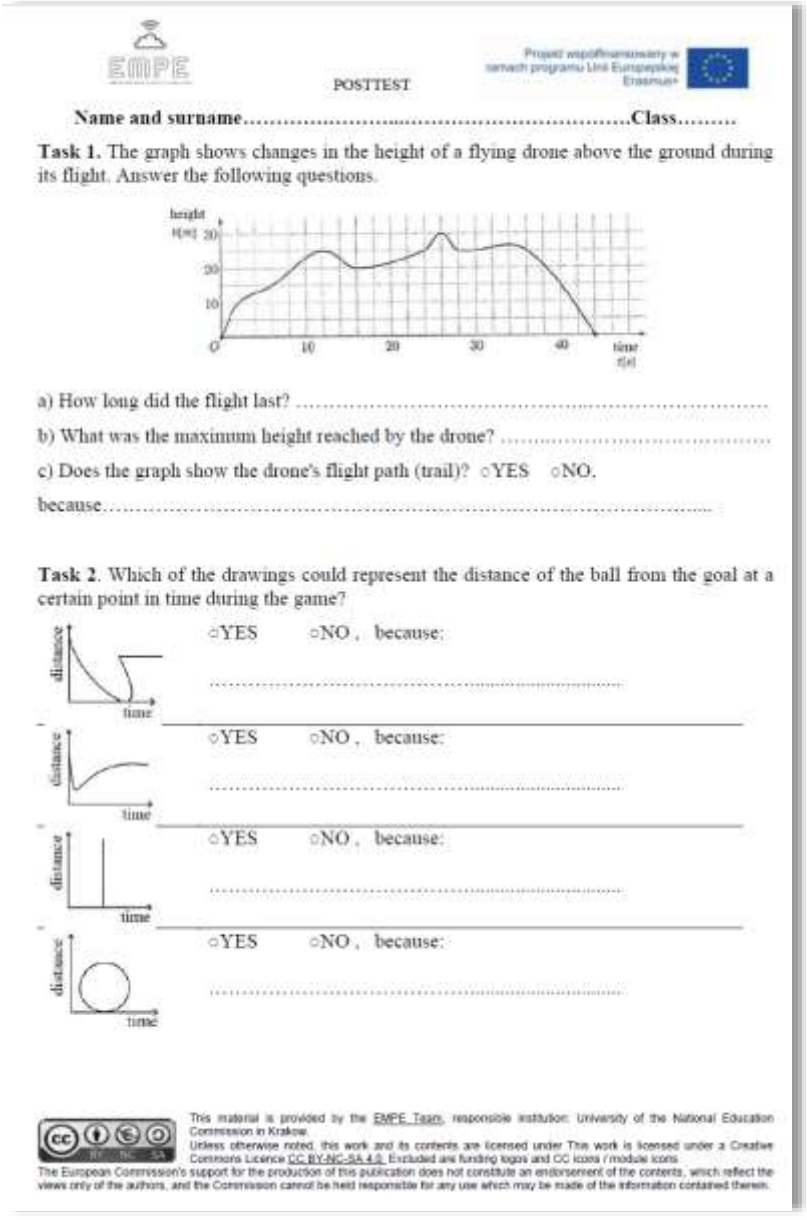
Then they answer the question below the table (Figure 4).

How can you tell when the movement is getting slower and when it is getting faster?

The answers to this question and their discussion in class conclude the teaching cycle.

POST-TEST

At the end of the teaching cycle, it is worth offering students a POST-TEST to assess their knowledge gain. The tasks are the same as in the PRE-TEST.



POSTTEST

Name and surname.....**Class**.....

Task 1. The graph shows changes in the height of a flying drone above the ground during its flight. Answer the following questions.

a) How long did the flight last? .....

b) What was the maximum height reached by the drone? .....

c) Does the graph show the drone's flight path (trail)? ○YES ○NO.

because.....

Task 2. Which of the drawings could represent the distance of the ball from the goal at a certain point in time during the game?

○YES ○NO , because:

.....

○YES ○NO , because:

.....

○YES ○NO , because:

.....

○YES ○NO , because:

.....