Projekt współfinansowany w ramach programu Unii Europejskiej Erasmus+

Mathematics
Cycle I Upper primary school
Understanding graphs and relationships

Topic 3	**We discover types of relationships analysing motion**
Duration	3 lessons (135 minutes)
Class/Age	The cycle is intended for students in the final years of primary school who are not familiar with the concept of functions, and we do not introduce this concept during the cycle (grade 7-8).
Objective	*The aim of this module is to develop an **intuitive** understanding of types of relationships and their graphs.* 1) Creating and interpreting graphs in the context of movement analysis at the intuitive level 2) Developing an understanding of graphs 3) Developing an intuitive understanding of unambiguous relationships between variables 4) Developing covariational reasoning
Description	Students create and examine graphs describing changes in distance over time using embodied experiments. During the lesson, students use the EMPE sensor together with the EMPE software. The sensor measures the distance to the nearest obstacle, and the software shows a real-time graph of changes in this distance over time. Students are involved in embodiment experiments by walking with the sensor and analysing the graphical interpretation of their movement. They have the opportunity to create and observe multiple graphs of different shapes, and they also perform reverse activities – they move in such a way as to reflect the movement shown in the graphs provided, and they interpret and analyse different movement graphs. The topic leads to an intuitive understanding of unambiguous relationships and their interpretation in the form of graphs. Due to the context of distance measurement and the discussion of "impossible graphs", students intuitively and practically discover the conditions of unambiguous relationships (which will be formalised as the definition of a function in later grades).

This material is provided by the EMPE Team, responsible institution: University of the National Education Commission in Krakow.

Teaching aids	- EMPE sensor with software - desktop computer or laptop with a web browser - projector screen - projector - work sheets for students

During the lesson, the teacher and students use the EMPE sensor with EMPE software developed as part of the EMPE project. Instructions for using the sensor can be found on the project website (https://empe.uken.krakow.pl).

TOPIC 3. Discovering types of relationships by analysing movement

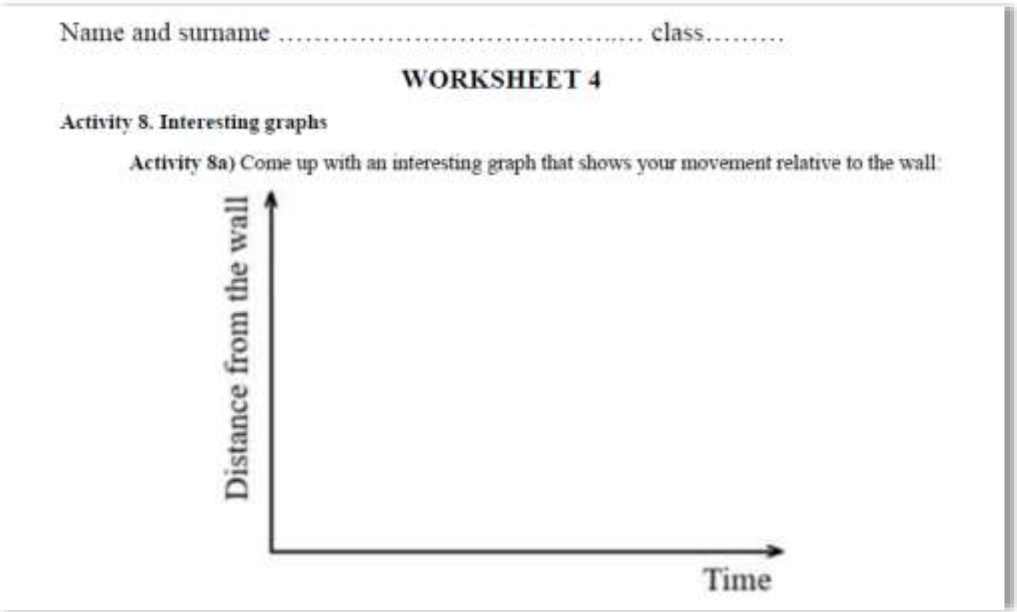
LESSON PLAN

We continue the numbering of activities from Parts 1 and 2 of Cycle I.

Activity 8. Interesting graphs

8a) Come up with an interesting graph

Independent work by students. Students complete Worksheet 4 by sketching an interesting graph showing their movement in relation to the wall:


Name and surname ……………………………..… class………

WORKSHEET 4

Activity 8. Interesting graphs

Activity 8a) Come up with an interesting graph that shows your movement relative to the wall:

Figure1. Worksheet 4, Activity 8a.

8b) Is the graph possible to complete?

Work in pairs. Students in pairs exchange worksheets. The students' task is to analyse their partner's graph.

Students are to answer the question on Worksheet 4:

Activity 8b. Swap your worksheets with the person sitting next to you.

Does this graph you received from your classmate show a movement that is possible to perform?

YES / NO / I DON'T KNOW

If YES, describe the movement in words;

if NO, write why;

if YOU DON'T KNOW, explain your doubts:

....

....

Figure2. Worksheet 4. Activity 8b

8c) We perform selected graphs

Work with the whole class.

If time and organisational circumstances allow, you can ask volunteers to perform the movements shown in their classmates' graphs using the motion sensor.

8d) Analysis of graphs that are impossible to perform

The teacher asks the students to redraw on the board the graphs (or parts of graphs) selected by him/her which, in their opinion, are impossible to perform.

Various suggestions from students appear on the board. Figure 3 shows an example of a worksheet with an impossible graph, and Figure 4 shows two boards from such lessons.

Figure3. An example worksheet with an impossible graph to complete

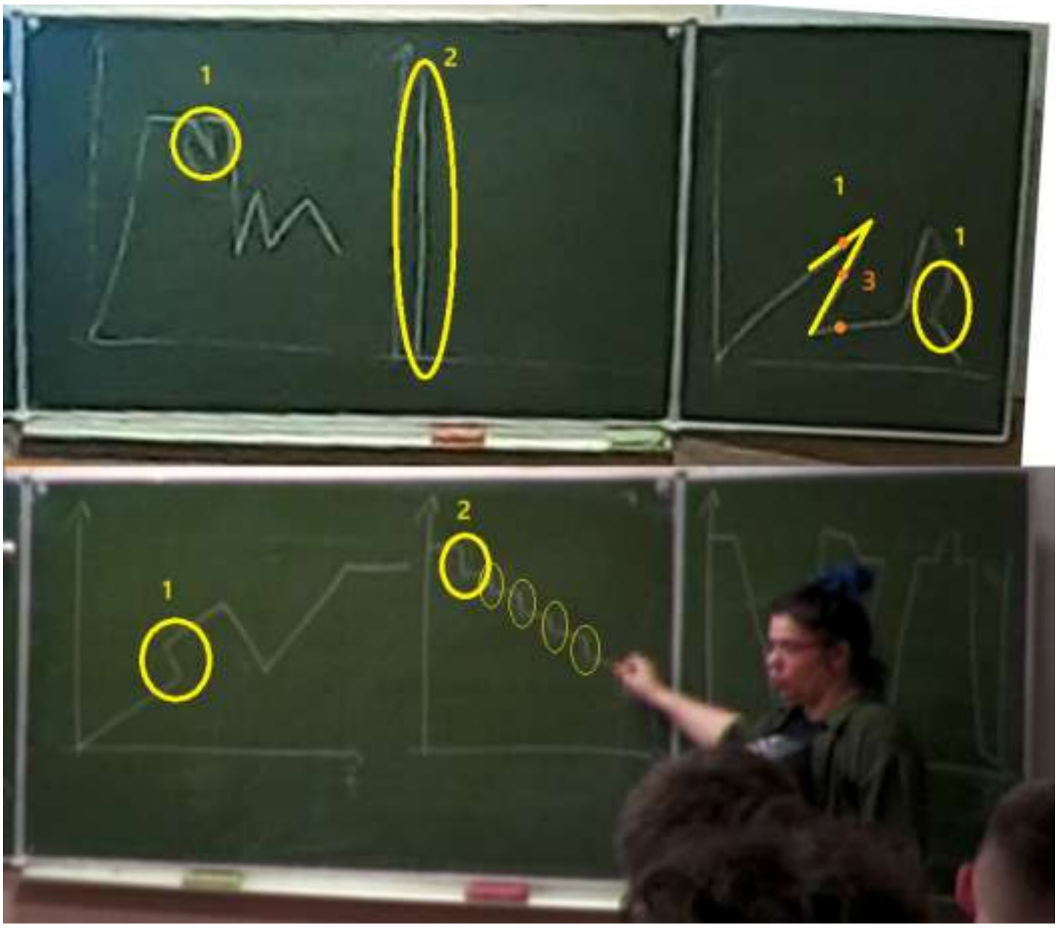


Figure4. Boards from the lesson during the analysis of impossible graphs

The teacher ensures that graphs appear showing that both conditions defining the function are not met in different graphs. We discuss at the board why the graph is impossible to obtain when moving with the sensor.

We take care to use natural language. There are four possible situations, which the pupils describe in different ways. Figure 4 shows selected situations labelled 1, 2 and 3.
In situations of type 1, students naturally say that it is impossible to go back in time (they emphasise drawing the graph "to the left"), in situations of type 2, students say that it is impossible to have many distances at one moment, sometimes they say that it is impossible to teleport in time or have multiple locations. Situation 3 can be discussed as a consequence of situation 1, as in Figure 4, or it may arise in a different context. It is important to discuss that we cannot obtain more than 1 distance from the wall at a single moment measured by a single sensor.

Possible vs. impossible situations in graphs – summary

We summarise the impossible situations.

Let us try to group the important reasons for impossible graphs.

The teacher displays graphs of types of impossible graphs (*Presentation*).

Figures 5 and 6 present an analysis of the first situation. This is usually perceived by students as going back in time (Figure 5):

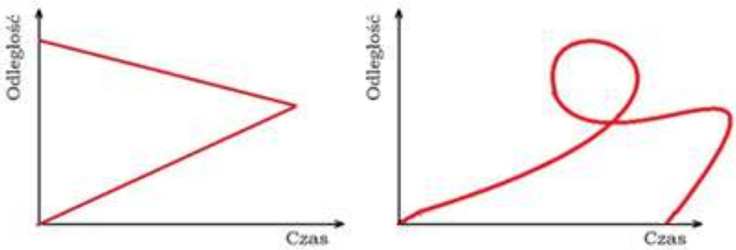


Figure5. Analysis of the first impossible situation

We draw students' attention to the fact that there would then be two or more distance measurements with one device at a given moment (Figure 6).

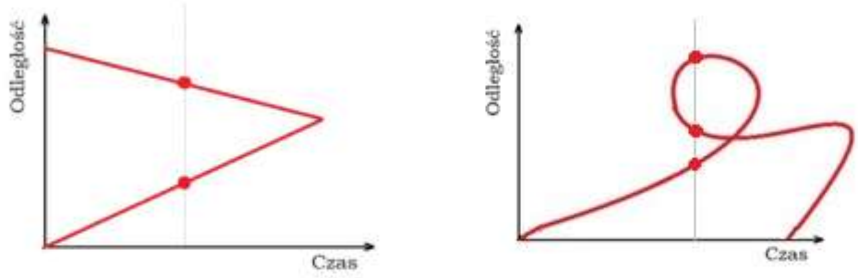


Figure6. Analysis of the first impossible situation – cont.

The next situation shown in Figure 7 shows an infinite number of measurement values at a single moment.

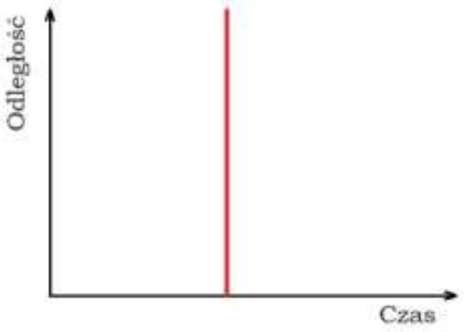


Figure7. Analysis of the second impossible situation

Comment. The vertical sections of the graph are qualitatively different for students, so we consider them separately. They do not show two or more values achieved at the same time, and the vertical line indicates continuity and movement. Students may equate such a graph with vertical upward movement. They often pay attention only to the dependent variable (y), ignoring changes in the independent variable (x). Therefore, this case should be discussed separately and only later should both cases be included in one condition.

The teacher asks the first key summary question:

- What condition must the graph meet in order to be possible? [Students should answer that **no more than one measurement result at a given moment**.]

The teacher asks the second key question:

- What about this graph (Figure 8)? We do not have more than one measurement at a given moment here, so is it possible to create?

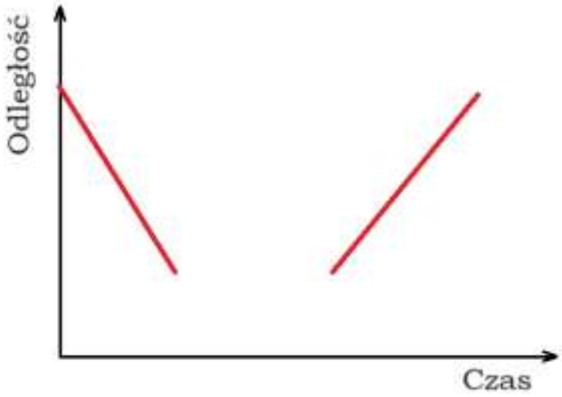


Figure8. Analysis of the last impossible situation

[Students should answer that **a break in the graph is not possible.** The teacher should ask why. If students have difficulty, the teacher may ask:

- What is our graph supposed to represent?

Students should explain that the graph shows changes in the distance between the sensor and a given obstacle over time. It is not possible for there to be no distance between the sensor and the obstacle, as the sensor will not disappear.

Comment. We also assume that the measuring sensor has not broken down. And even if it did break down, it is important to remember what it measures (the distance from the nearest obstacle), so this distance still exists and can be measured by other means.

The teacher summarises by asking the question:

- How can we briefly describe the condition for the graph to be feasible?

[Students should answer that **there must always be exactly one** measurement **value** at a given moment.]

Comment. This condition is an intuitively formulated condition defining a functional relationship in the context of measuring distance – for each argument (point in time) there is exactly one value (measurement).

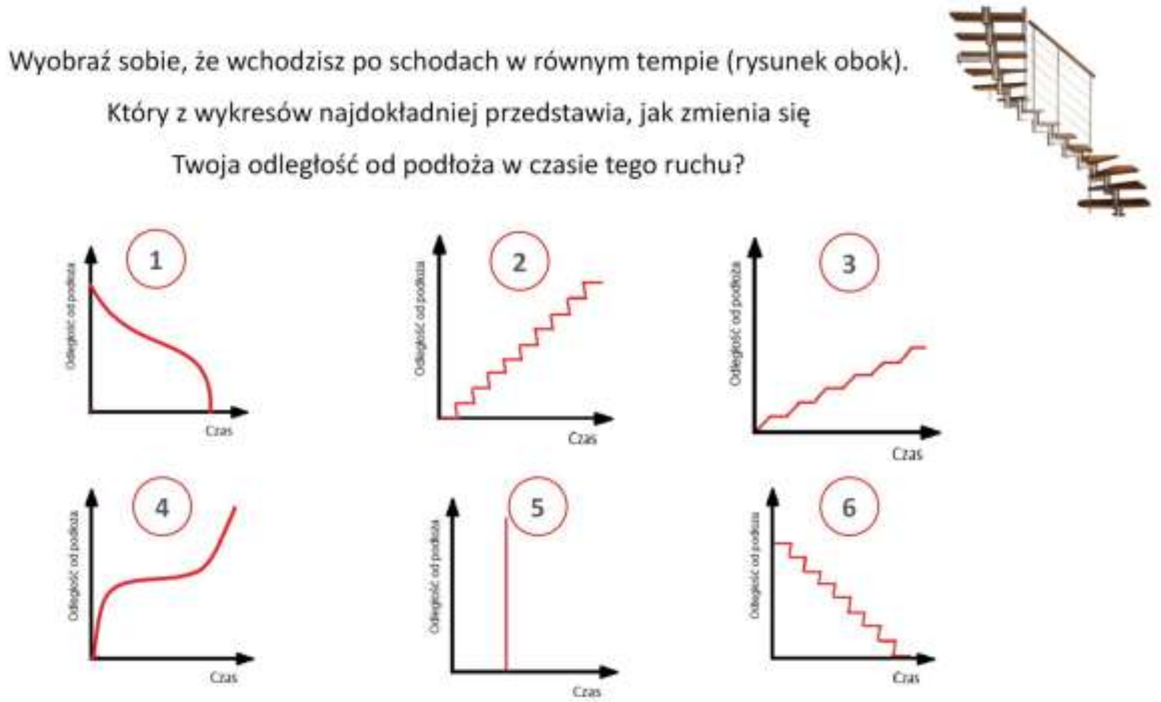
Relationships that meet these conditions are called unambiguous.

At the end, the teacher may mention that in older classes, students will learn that relationships that meet this condition are called *functions*.

Staircase task - recall

We recall the task from the previous lesson. The teacher displays a slide with the task.

- Knowing how to recognise impossible graphs, how could the staircase task be solved?



[The pupils answer that graphs 2, 5 and 6 are impossible, and in graphs 1 and 6 the distance from the ground decreases, so they also do not correspond to the content of the task. By process of elimination, there are two that may be correct – graphs 3 and 4.]

Activity 9. Come up with a movement that can be represented by a horizontal line graph

The teacher displays the slide and the instruction.

- Come up with a movement in which the measured distance is constant:

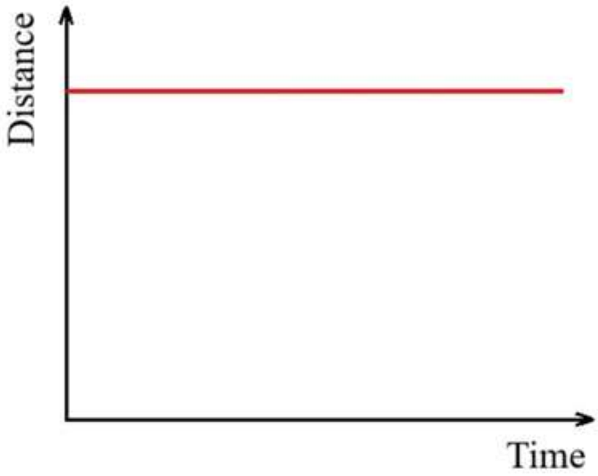


Figure9. Come up with a movement described by this graph.

The students first think about the movement individually. Then they exchange and discuss new ideas in pairs. After a while, when the pair work phase is no longer effective, the teacher leads a discussion with the whole class, collecting different ideas and asking about other types of movement. The pupils perform the movement they have devised, using the sensor to measure it and verify their answers.

Comment. Students first see the possibility of standing still. However, after a while, other ideas should emerge. Below are some examples that came up during the lessons.

- *Measuring the distance from the wall while jumping with a sensor in place*
- *Measuring the distance from the wall while moving sideways, at the same distance from the wall*
- *Measuring the distance from the ceiling/floor while walking freely around the room – in this case, refer to the PRETEST task about the drone and the fact that the drone could move horizontally and maintain a constant distance from the ground (Fig. 10)*

Figure10. Measuring the distance from the ceiling/floor.

- *Measuring the distance from two people moving simultaneously (Fig. 11)*

Figure11. Measuring the distance from two people moving simultaneously.

- *Measuring the distance from a person standing in the centre of a circle while moving in a circle (Fig. 12)*

Figure12. Distance when moving in a circle.

- *Measuring the distance from people standing in a circle by a person spinning around their own axis (Fig. 13)*

Figure13. Distance from people standing in a circle by a person spinning around their own axis.

In the context of various statements made by students, interesting ideas may arise that require further investigation. For example, in one class, a student suggested measuring the distance from the intersection of the diagonals in a square to the sides of the square. A model was made in which the sensor was placed on a swivel chair (Figs. 14 and 15) and the graph was checked:

Figure14. Model for measuring the distance between the intersection point of the diagonals of a square and the sides of the square (1)

Figure15. Model for measuring the distance between the intersection point of the diagonals of a square and the sides of the square (2)