Projekt współfinansowany w ramach programu Unii Europejskiej Erasmus+

Mathematics
Cycle I Upper primary school
Understanding graphs and relationships

The scenario includes a description of adjustments for pupils with special educational needs (A, ASD, ID)

Topic 1	**We create and interpret graphs describing motion**
Duration	3 lessons (135 minutes)
Class/Age	The teaching cycle is aimed at pupils in the final years of primary school who are not familiar with the concept of functions, and we do not explicitly introduce this concept during the cycle (grade 7-8).
Type of adjustments	- aphasia (A), - autism spectrum disorder (ASD) - mild intellectual disabilities (ID)
Objective	*The aim of this module is to develop an **intuitive** understanding of types of relationships and their graphs.* 1) Creating and interpreting graphs in the context of motion analysis at the intuitive level 2) Developing an understanding of graphs 3) Developing an intuitive understanding of unambiguous relationships between variables 4) Developing covariational reasoning
Description	Students create and examine graphs describing changes in distance over time using embodied experiments. During the lesson, students use the EMPE sensor together with the EMPE software. The sensor measures the distance to the nearest obstacle, and the software shows a real-time graph of changes in this distance over time. Students are involved in embodiment experiments by walking with the sensor and analysing the graphical interpretation of their movement. They have the opportunity to create and observe multiple graphs of different shapes, and they also perform reverse activities – they move in such a way as to reflect the movement shown in the graphs provided, and they interpret and analyse different movement graphs.

This material is provided by the EMPE Team, responsible institution: University of the National Education Commission in Krakow.

Teaching aids	- EMPE sensor with software - desktop computer or laptop with a web browser - projector screen - projector - work sheets for students

During the lesson, the teacher and students use the EMPE sensor with EMPE software developed as part of the EMPE project. Instructions for using the sensor can be found on the project website (https://empe.uken.krakow.pl).

The scenario includes a description of adjustments for pupils with special educational needs, highlighted in green. Adjustments have been made for working with pupils with **aphasia (A)**, **autism spectrum disorder (ASD)** and **mild intellectual disability (ID)**.

TOPIC 1. We create and interpret graphs describing motion

LESSON PLAN

PRE-TEST

At the beginning of the lesson cycle, we can ask students to individually and in writing complete a two-task PRETEST. Its purpose is to test their intuitive understanding of graphs.

Students with autism spectrum disorder (ASD) complete the pretest (ASD).pdf
Students with aphasia complete the pretest (A).pdf.
Students with mild intellectual disabilities complete the pretest (ID).pdf

In each version of the pretest, the content of each task fits on one page – we do not move it from page to page. The student must have constant visual contact with the graph. The font and drawings are enlarged to facilitate the students' visual perception of both the image and the verbal content. Some content is written in bold, while other content is underlined to draw the student's attention to the most important information. In pre-tests for students with aphasia and mild intellectual disabilities, in task 1, more points should be marked and labelled on both axes of the coordinate system. Line spacing – at least 1.5. It would be good to print the test on both sides of the page.

Possible difficulties when working with the pre-test;

- autism spectrum disorder (ASD):

- in order to follow the instructions precisely, the student may need more time than other students to complete the tasks,
- the student may ask additional questions,

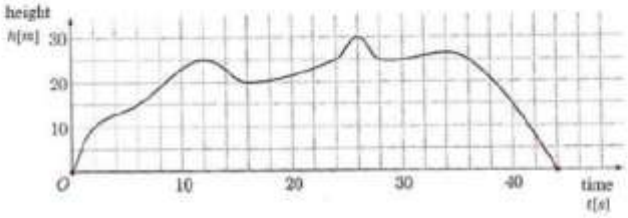
- aphasia (A):
  - the student may have difficulty reading and understanding the content of the tasks on their own and will therefore need the teacher's help,
  - working slowly, the student may need more time than other students to complete tasks,
- mild intellectual disability (ID):
  - expressing movement in the form of a graph is based on thought processes, which in the case of this student are at a significantly lower level of functioning, and may require considerable assistance from the teacher,
  - the student may have difficulty reading and understanding independently and will need the teacher's support if necessary,
  - the student may have difficulty formulating thoughts independently in writing and will need the teacher's support if necessary,
  - control questions must be used to ensure that the pupil understands the content of the task to be performed,
  - The teacher's assistance will necessitate an extension of the student's working time.

The Pretest tasks are as follows:

PRETEST

Name and surname……Class……

Task 1. The graph shows changes in the height of a flying drone above the ground during its flight. Answer the following questions.



a) How long did the flight last? ……

b) What was the maximum height reached by the drone? ……

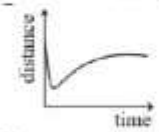
c) Does the graph show the drone's flight path (trail)? ○YES ○NO,

because……

Task 2. Which of the drawings could represent the distance of the ball from the goal at a certain point in time during the game?

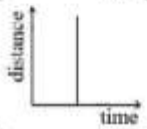
○YES ○NO , because:

……



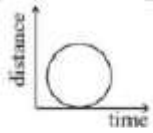
○YES ○NO , because:

……



○YES ○NO , because:

……



○YES ○NO , because:

……

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The teacher collects the Pretest and will refer to it in the subsequent stages of the lesson cycle.

Activity 1. Drama and experiment

1a) Drama. Formulating hypotheses

The teacher reads the movement scenario described in Worksheet 1:

> *At first, I stand still for a moment.*
>
> *Then I walk at a steady, moderate pace towards the wall, and then at the same pace away from the wall. I stand still for a moment. Then I walk at a steady, fast pace towards the wall, and then at the same pace away from the wall.*
>
> *I stand still for a moment.*
>
> *Finally, I walk at a steady, slow pace towards the wall, and then at the same pace, I move away from the wall.*
>
> *At the end, I stand still for a moment.*

The content of the instruction should be visible on the projector at all times so that students with reduced auditory perception have the opportunity for visual perception, which will help them understand the course of the experience.
Students with autism spectrum disorder (ASD), aphasia and mild intellectual disabilities will need clarification through the teacher's demonstration of the phrases: walk at a moderate pace, walk at a steady fast pace, walk at a steady slow pace.

Then, the movement described above is performed by a selected student or teacher.

It is a good idea to involve pupils with a special educational needs certificate in the task so that they have a chance to practise this movement.

Possible difficulties during Activity 1:

- autism spectrum disorder (ASD):
  - in order to precisely follow the instructions in the movement scenario, the pupil may need to be told (spoken) the specific number of steps to be taken or the point to be reached (this can be marked on the floor, e.g. with adhesive tape). The number of steps or the indicated point will, of course, depend on the size of the room in which the lesson takes place.
  - the student may expect (e.g. by asking questions) additional instruction,

- Aphasia (A):
  - the student may have difficulty understanding the content of the movement scenario and will therefore need the help of the teacher, who will read the scenario and perform the desired movement, and only after this help will the student perform the task,
- mild intellectual disability (ID):
  - similar to a student with aphasia, they may have difficulty understanding the content of the movement scenario and will therefore need the help of a teacher who will simultaneously read the scenario and perform the desired movement, and only after this assistance will the student perform the task,
  - the pupil may need to be told (spoken) the specific number of steps to be taken or the point to be reached (this can be marked on the floor, e.g. with adhesive tape). The number of steps or the point indicated will, of course, depend on the size of the room in which the lesson takes place.

It is worth starting the walk from the back of the room and moving towards the board, because in this arrangement, when the pupil walks from the back of the room to the front, we will elicit typical mistakes (see Figure 2 on the left) and it will be possible to work on them later in the lesson.

After completing the movement (performing the drama), the teacher distributes Worksheets 1 (Fig. 1) to the pupils.

Students with autism spectrum disorder (ASD) complete WORKSHEET 1 (ASD).
Students with aphasia fill in WORKSHEET 1 (A).
Students with mild intellectual disabilities complete WORKSHEET 1 (ID).

The students' task is to make their first attempt at sketching the shape of a graph showing the changes in distance from the wall during this movement.

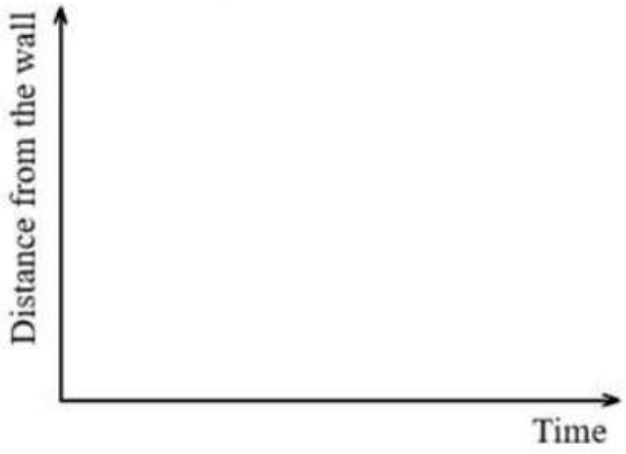
Name and surname ..... class.....

WORKSHEET 1

Activity 1. Sketch what the graph of my distance from the wall will look like, taking into account all stages of my movement:

I begin by standing still for a short moment. Then I walk toward the wall at a steady, moderate pace, and after reaching a point near the wall, I walk away from it at the same pace. Again, I stop and stand still for a moment. Next, I walk toward the wall at a steady, fast pace, and then walk away from the wall at the same fast pace. I pause and stand still again. Finally, I walk toward the wall at a steady, slow pace, and then walk away from it at the same slow pace. At the end, I stand still for a moment once more.

Your graph – first attempt:



1 , Worksheet 1, Activity 1

Possible difficulties during the activity:

- autism spectrum disorder (ASD):
  - the student may expect additional instruction because they may feel confused by such freedom of action,
- Aphasia (A):
  - the student may have difficulty reading and understanding the movement scenario and will therefore need the teacher's help in reading the subsequent stages of the movement,

- the student may need information about what the individual arrows (axes of the system) mean,
- it may be necessary to show the student where to start drawing the graph,

- mild intellectual disability (ID):
  - similar to a student with aphasia, they may have difficulty reading and understanding the movement scenario and will therefore need the help of a teacher who will read the subsequent stages of the movement to them,
  - in the case of difficulties with reading comprehension, it may be necessary to work in stages. The teacher reads a fragment of the movement scenario and the student draws it on the graph. Then the teacher reads the next fragment and the student proceeds as before. The sequence of actions is repeated until the end of the scenario.
  - The student will most likely need information about what the individual arrows (axes of the system) mean.
  - It will most likely be necessary to show the student where to start the graph.

1b) Discovering how the sensor works

We proceed to manipulate the sensor. The teacher shows the sensor, opens the software and the graph, displays it on the projector and starts the measurement. They direct the sensor in different directions. The pupils observe how the graph is created in the application at this time.

Ensure that the teacher and their actions are clearly visible from every part of the classroom.

The teacher asks the question:

- What can you say about this graph? (What are the descriptions of the coordinate axes? What is being measured?)

We wait for the pupils' answers, which will be:

- the sensor measures the distance to the nearest obstacle it encounters,

- the graph shows changes in the distance of the sensor from the nearest obstacle over time.

During the discussion, the teacher should encourage (but not force) students with autism spectrum disorder (ASD), aphasia and mild intellectual disabilities to participate actively. Conditions should be created for students to speak and present their ideas: give them more time to speak, ensure that they are not interrupted by other students. This will not only allow them

to be actively involved in the lesson, but also enable the teacher to make sure that the students understand the content being taught and to correct any mistakes.

Possible difficulties during work;

- autism spectrum disorder (ASD):
  - if a student has difficulty thinking in terms of cause and effect, they will be a passive participant in the discussion,
  - by making specific mental associations, they may express themselves in a way that is surprising to the teacher,
  - the student may ask additional questions,
- aphasia (A):
  - the student may have difficulty verbalising their thoughts correctly and should be helped, e.g. by suggesting appropriate words,
  - having problems drawing conclusions, their statements may deviate so significantly from what is expected that in order to properly guide both the course of the discussion and the student's train of thought, it will be necessary to calmly and tactfully, but firmly, correct their statements,
- Mild intellectual disability (ID):
  - expressing movement in the form of a graph is based on thought processes, which in the case of this student are at a significantly lower level of functioning, and their statements may be completely inadequate to the topic being discussed, which will necessitate a similar approach as in the case of a student with aphasia; calm and tactful, but firm correction of their statements.
  - the student may have difficulty formulating their thoughts independently and orally and, if necessary, will need the teacher's support in the form of, for example, prompting words.

1c) Performing the experiment with the sensor. Verifying hypotheses

We perform the experiment described at the beginning of the lesson, this time using the sensor. The selected student performs the described movement independently.

The teacher reads the instructions while the exercise is being performed.

Comment. Students should be instructed to hold the sensor in the same position (e.g. close to their body) and not to move it, especially in the front-back directions. It is worth repeating the experiment several times with different students.

A correctly constructed graph should have significantly different slopes of the straight lines:

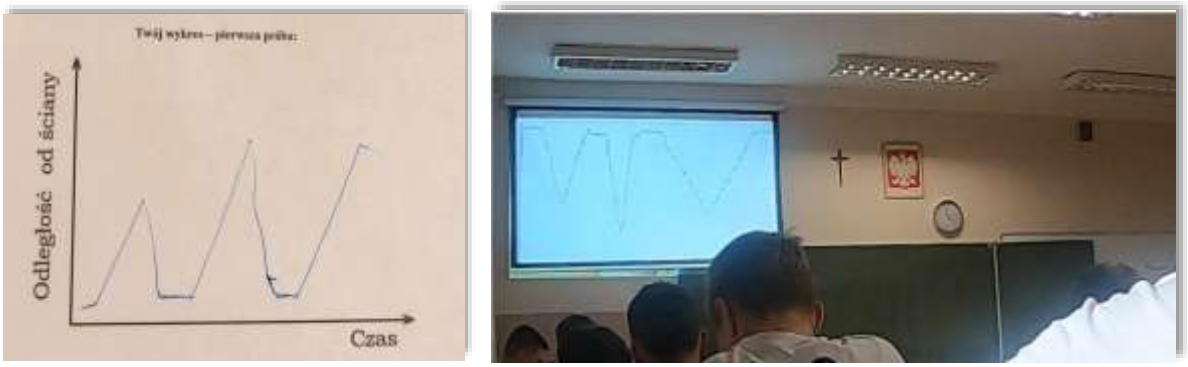


Figure2 . On the left – a typical incorrect prediction by a student. On the right – a graph produced by the sensor.

The students redraw the correct graph.

If possible, for students with autism spectrum disorder (ASD), aphasia, and mild intellectual disabilities, it would be beneficial to print out the graph created during the experiment so that they can paste it into their notebooks.

Possible difficulties during the activity:

- autism spectrum disorder (ASD):
  - if the student does not have graphomotor difficulties, they should not have any major difficulties with this task,
  - if graphomotor difficulties occur, the teacher's help will be necessary,
- Aphasia (A):
  - as the student may make significant mistakes when redrawing the graph, it is necessary for the teacher to monitor their actions and provide assistance if necessary,
- mild intellectual disability (ID):
  - similar to a student with aphasia, they may make significant mistakes when redrawing the graph, and supervision and assistance from the teacher will be necessary.

Students answer the question below the graph on their own: *What do you notice?*

Since we are interested in all possible answers from students, we deliberately ask an imprecise question. With such a high degree of generality, pupils with aphasia and intellectual disability may not be able to answer this question. However, if they decide to answer (in writing), they should be helped to formulate their thoughts by suggesting appropriate words or asking supporting questions.

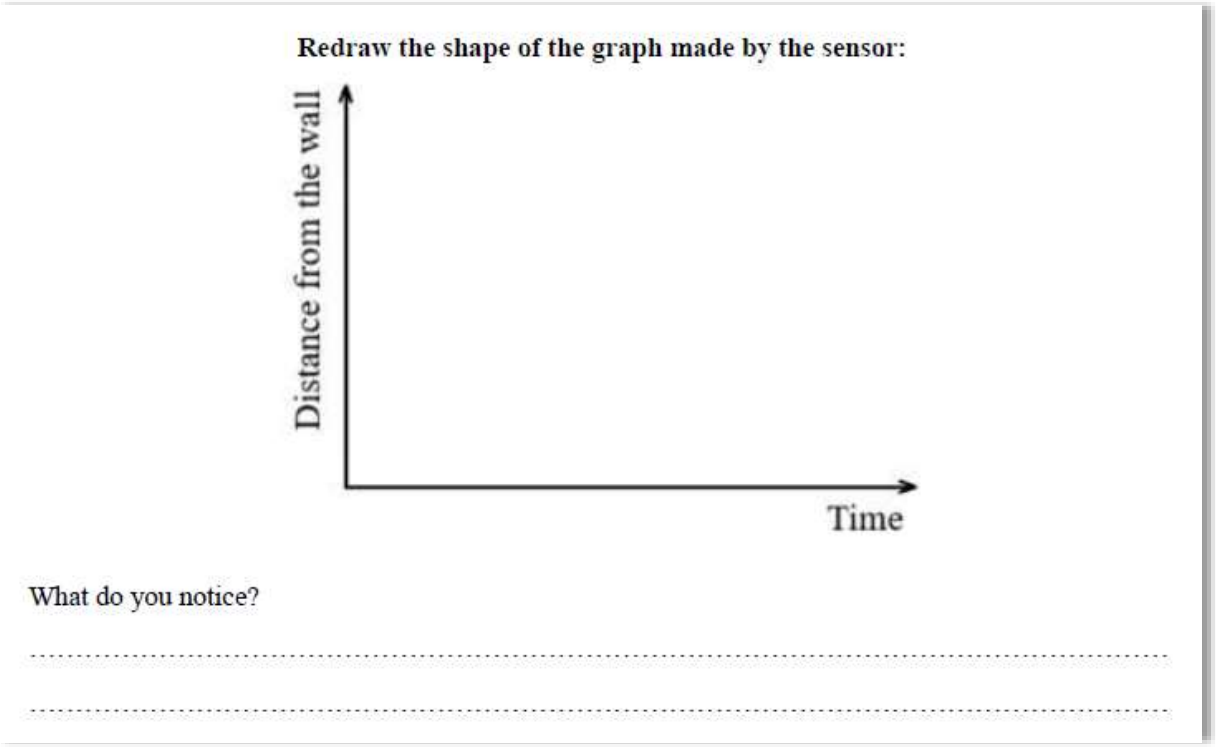


Figure3 . Worksheet 1, Activity 1 (cont.).

1d) Analysis of the graph

Please read the students' statements.

> *Comment: A typical mistake is to draw the graph following the trajectory of movement to the board, as in Fig. 2 (left). We refer to this mistake in the discussion and analyse it with the pupils, explaining why such a graph is drawn incorrectly.*

The teacher then asks the students questions to facilitate the analysis of the entire graph, for example:

- Why is the graph horizontal at the beginning? (we are standing, so the distance is the same)

This horizontal section should be indicated on the graph. Students with autism spectrum disorder (ASD), aphasia and mild intellectual disabilities may be asked to mark this section of the graph with a selected colour. Under the graph, the statement: the distance from the wall does not change should be underlined in the same colour. After completing the task, check that it has been done correctly.

- How does the distance between the sensor and the wall change during movement towards the wall? (the distance decreases)

Indicate this section on the graph. Students with autism spectrum disorder (ASD), aphasia and mild intellectual disability with aphasia may be asked to mark this section of the graph with a selected (second) colour. Under the graph, use the same colour to underline the statement: the distance from the wall decreases. After completing the task, check that it has been done correctly.

- How does the distance between the sensor and the wall change when moving away from the wall? (the distance increases)

Indicate this section on the graph. Students with autism spectrum disorder (ASD), aphasia and mild intellectual disability may be asked to mark this section of the graph with a selected (third) colour. Under the graph, underline the statement: the distance from the wall increases with the same colour. After completing the task, check that it has been done correctly.

- How can we tell from the graph when we were walking fast? (greater slope of the line – more vertical segments of the line, distance covered in a short time)

Indicate this section on the graph. Students with autism spectrum disorder (ASD), aphasia, and mild intellectual disabilities may be instructed to mark this section of the graph with a selected (fourth) colour. Underneath the graph, underline the statement: we walked quickly with the same colour. After completing the task, check that it has been done correctly.

- How can we tell from the graph when we were walking slowly? (smaller slope of the line – more horizontal segments of the line, distance covered in a long time)

Indicate this section on the graph. Students with autism spectrum disorder (ASD), aphasia and mild intellectual disabilities may be asked to mark this section of the graph with a selected (fifth) colour. Underneath the graph, underline the statement: we walked slowly in the same colour. After completing the task, check that it has been done correctly.

For subsequent questions, the teacher points to the relevant parts of the graph.

- Was the break in movement the same length each time?

- Was the break in movement the same distance from the board each time?

- Why are there different slopes on the graph? (because we walk at different speeds)

- If we walk very fast, will the graph be vertical?

- If we walk very slowly, will the graph be horizontal? (as the pace increases, the graph becomes more and more vertical)

Comment: It is worth paying attention to distractors and imperfections in the graph. For example, in Fig. 4, you can see how the person holding the sensor stumbled while moving slowly towards the board, which is marked in red in the photograph.

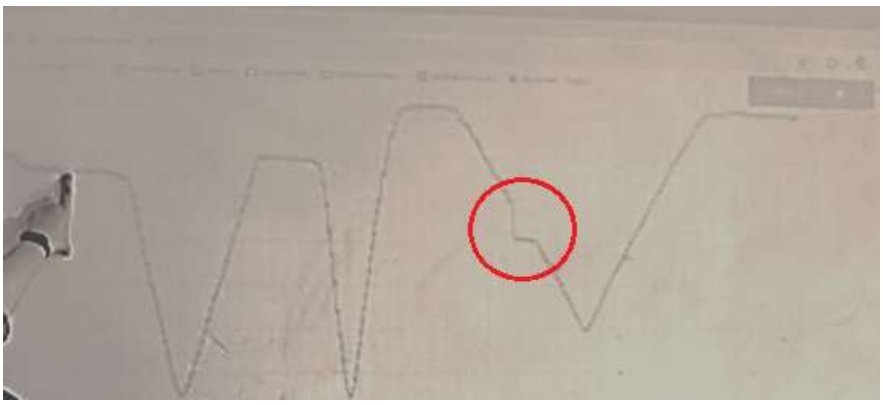


Figure4 . Analysis of distractors on the graph.

Activity 2. Slowly – quickly at a steady pace TOWARDS the wall

Independent work by students. Students complete the Worksheet (p. 2):

Two people walk TOWARDS the wall, starting at the same distance from the wall, both walking at the same pace but one slowly and the other quickly. Sketch in one coordinate system what the graphs of both movements will look like.

Use two different colours to draw both graphs. Ensure that students with autism spectrum disorder (ASD), aphasia and mild intellectual disabilities perform the task (graph) well by providing ongoing supervision and additional instruction.

Person 1 walks towards the wall slowly – line named s

Person 2 walks towards the wall quickly – line named q

In your own words, how can you tell from the chart when the movement towards the wall was slow and when it was fast?

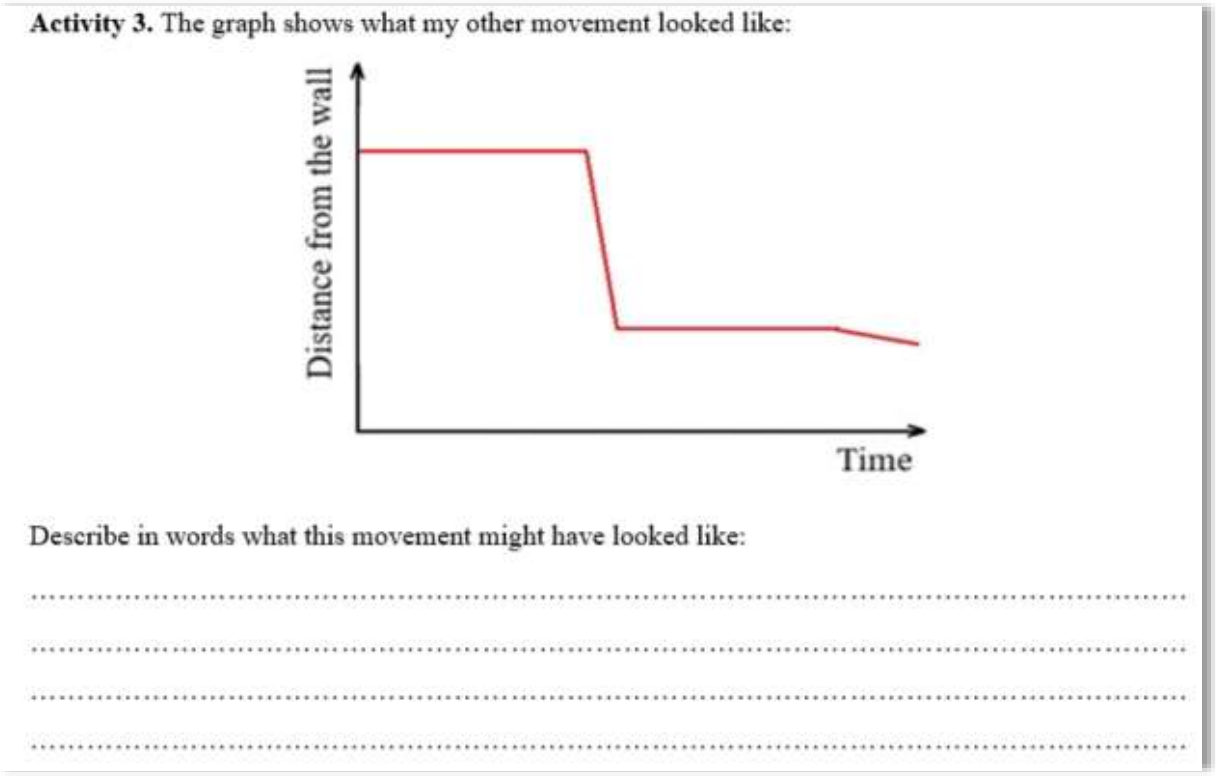
Students with aphasia and students with mild intellectual disabilities should try to complete this task on their own. However, it may turn out that the sheet will be blank because they will not be able to analyse the chart without the teacher's help.

- Students with aphasia may have difficulty expressing their thoughts in writing.
- For pupils with mild intellectual disabilities, the task is difficult because it requires them to perform a series of mental operations and formulate oral statements at the same time.

Selected pupils read out their statements.

Activity 3. Interpretation and description of the graph

The pupils are asked to perform the reverse activity, which consists of describing in words the possible movement shown in the graph:



5 . Worksheet 1, Activity 3.

Students with autism spectrum disorder (ASD), aphasia, and mild intellectual disabilities may have difficulty interpreting the graph on their own. They will need help and guidance. You can suggest that they mark the individual parts of the graph with consecutive numbers (1–4) and then use these numbers to describe the movement shown. It will be helpful to include a plan of the subsequent stages of activities on the worksheet. This will be particularly helpful for pupils with aphasia and pupils with mild intellectual disabilities, for whom you can additionally prepare a glossary of words and phrases that may be useful in describing the movement shown in the graph.

Activity 4. Slowly – quickly at a steady pace AWAY FROM the wall

Independent work by students. Students complete Worksheet 2 (p. 3):

Two people walk FROM the wall, starting at the wall and ending the movement at the same distance from the wall, both walking at the same pace but one slowly and the other quickly. Sketch in one coordinate system what the graphs of both movements will look like.

Person 1 walks slowly away from the wall – line named s
Person 2 walks quickly away from the wall – line named q

As in Activity 2, use two different colours to draw both graphs. Ensure that students with autism spectrum disorder (ASD), aphasia and mild intellectual disabilities perform the task (graph) well through ongoing monitoring and additional instruction.

In your own words, how can you tell from the graph when the movement towards the wall was slow and when it was fast?

As in Activity 2:

Students with aphasia and students with mild intellectual disabilities should try to complete this task on their own. However, they may find that the sheet is blank because they are unable to analyse the graph without the teacher's help.

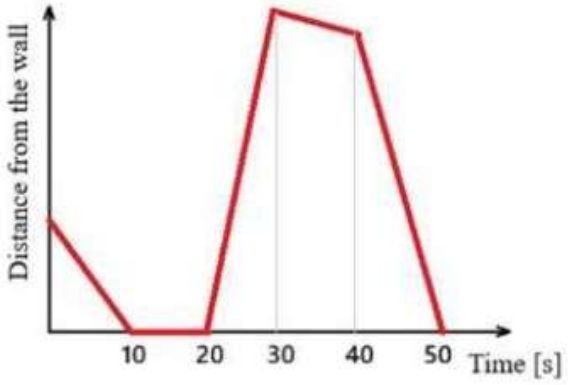
- Students with aphasia may have difficulty expressing their thoughts in writing.

for students with mild intellectual disabilities, the task is difficult because it requires them to perform a series of mental operations and formulate oral statements at the same time.

Activity 5. Summary – Graph analysis

Students fill in the table by interpreting the movement chart:

Activity 5. Based on the graph describing the movement, complete the table (write a word or tick ✓).



Time interval	0-10 [s]	10-20 [s]	20-30 [s]	30-40 [s]	40-50 [s]
Slow/ Fast /Moderate					
TOWARDS / FROM wall					
Fastest					
Does not change distance					

How do you know when the movement was fastest?

.....

.....

Figure6 . Worksheet 2, Activity 5.

Students with autism spectrum disorder (ASD), aphasia and mild intellectual disabilities may have difficulty interpreting the graph and completing the table correctly on their own. They will need help and guidance not only through additional verbal instruction, but also through detailed written instructions. It will be helpful to provide detailed information that the first and second rows of the table should be filled in with the selected word, and the third and fourth rows with a V.

When working with the table, students with aphasia and students with mild intellectual disabilities will benefit from numbering the rows and covering the rows that they are not

analysing at a given moment. However, constant supervision and possible assistance from the teacher will be necessary.

At the end of the lesson, make sure that the pupils have taken proper notes.